7.1 $\log\left(\frac{S(t)}{S(0)}\right) \sim$ follows $\begin{matrix}(r-\frac{\sigma^2}{2})t \\ \sigma^2 t\end{matrix}$

$\log(S(t_1)) - \log S(0) \sim \begin{matrix}(r-\frac{\sigma^2}{2})t_1 \\ \sigma^2 t_1\end{matrix}$

$\log(S(t_2)) - \log S(0) \sim \begin{matrix}(r-\frac{\sigma^2}{2})t_2 \\ \sigma^2 t_2\end{matrix}$

thus $\log S(t_1) - \log S(t_2) \sim \begin{matrix}(r-\frac{\sigma^2}{2})(t_1-t_2) \\ \sigma^2(t_1-t_2)\end{matrix}$

$= \log\left(\frac{S(t_1)}{S(t_2)}\right)$

(a) $\log\left(\frac{S_d(n)}{S_d(n-1)}\right) \sim \sigma \cdot \sqrt{\frac{1}{365}}$

(b) $\log\left(\frac{S_m(n)}{S_m(n-1)}\right) \sim \sigma \sqrt{\frac{1}{12}}$

7.2 The option will be exercised if and only if the price at $t=\frac{1}{3}$ is bigger than 42.

$$P(S(\tfrac{1}{3}) > 42) = P\left(\frac{S(\frac{1}{3})}{S(0)} > \frac{42}{40}\right)$$

$$= P\left(\log\left(\frac{S(\frac{1}{3})}{S(0)}\right) > \log\frac{42}{40}\right)$$

$$= P(X > \log 1.05) \qquad X = \log\frac{S(\frac{1}{3})}{S(0)}$$

here X is N.R.V. with $\mu t = 0.12 \cdot \frac{1}{3} = 0.04$

and $\sigma^2 t = 0.24^2 \cdot \frac{1}{3}$

thus $P(X > \log 1.05) = P\left(\frac{X - 0.04}{\sqrt{0.24^2 \cdot \frac{1}{3}}} > \frac{\log 1.05 - 0.04}{\sqrt{0.24^2 \cdot \frac{1}{3}}}\right)$

$$= P\left(Z > \frac{\log 1.05 - 0.04}{\sqrt{0.24^2 \cdot \frac{1}{3}}}\right) = 1 - \Phi(0.0634)$$

$$= 1 - 0.5253 = 0.4747$$

7.3 $C = S\Phi(w) - ke^{-rt}\Phi(w-\sigma\sqrt{t})$

$= 40\Phi(w) - 42e^{-0.08\cdot\frac{1}{3}}\Phi(w - 0.24\cdot\sqrt{\frac{1}{3}})$

$$w = \frac{rt + \frac{\sigma^2 t}{2} - \log(\frac{K}{S})}{\sigma\sqrt{t}}$$

$$= \frac{0.08\cdot\frac{1}{3} + \frac{0.24^2\cdot\frac{1}{3}}{2} - \log(\frac{42}{40})}{0.24\cdot\sqrt{\frac{1}{3}}} = -0.0904$$

$C = 40\Phi(-0.0904) - 42e^{-0.08\cdot\frac{1}{3}}\Phi(-0.2289)$

$= 40 * 0.4639 - 42e^{-0.08\cdot\frac{1}{3}} * 0.4094$

$= 1.8137$

7.4 by put-call parity

$P = C - S(0) + ke^{-rt}$

$= S(0)\Phi(w) - ke^{-rt}\Phi(w-\sigma\sqrt{t}) - S(0) + ke^{-rt}$

$S(0) = 105. \quad K = 100. \quad r = 0.1. \quad t = \frac{1}{2}. \quad \sigma = 0.3$

then $w = 0.57167 \qquad \Phi(w) = 0.7163$

$w - \sigma\sqrt{t} = 0.359635 \qquad \Phi(w-\sigma\sqrt{t}) = 0.6404$

$P = 105\cdot 0.7163 - 100e^{-0.1\cdot\frac{1}{2}} * 0.6404 - 105 + 100e^{-0.1\cdot\frac{1}{2}}$

$= \ldots$

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7.5 (a) $P\left(\frac{S(0.5)}{S(0)} < 0.9\right) = P\left(\log\frac{S(0.5)}{S(0)} < \log 0.9\right)$

$\log\left(\frac{S(0.5)}{S(0)}\right)$ is N.R.V. with $\mu \cdot 0.5 = 0.06 * 0.5$

$\sigma^2 \cdot 0.5 = 0.3^2 * 0.5$

thus $P\left(\log\left(\frac{S(0.5)}{S(0)}\right) < \log 0.9\right) = P\left(\frac{\log\left(\frac{S(0.5)}{S(0)}\right) - 0.03}{\sqrt{0.045}} < \frac{\log 0.9 - 0.03}{\sqrt{0.045}}\right)$

$= P(Z < -0.638)$

$= \Phi(-0.638)$

$= 1 - \Phi(0.638) = 0.2617$

(b) $r - \frac{\sigma^2}{2} = 0.05 - 0.045 = 0.005$

Repeat (a)

we can find

$P\left(\frac{S(0.5)}{S(0)} < 0.9\right) = 1 - \Phi(0.508) \approx 0.3057 = P.$

thus Return of this investment is

$$\begin{cases} 100e^{-rt} - A & \text{if price } \frac{S(0.5)}{S(0)} < 0.9 \\ -A & \text{otherwise} \end{cases}$$

so $E[\text{Return}] = 0 \Rightarrow P \cdot (100e^{-rt} - A) + (1-P)(-A) = 0$

$\Rightarrow A = P \cdot 100e^{-rt} = 100e^{-0.025} \cdot P\left(\frac{S(0.5)}{S(0)} < 0.9\right)$

$= 100e^{-0.025} \cdot 0.3057$

7.6 ⓐ $C = S\Phi(w) - Ke^{-rt}\Phi(w-\sigma\sqrt{t})$

$= 95\Phi(w) - 100e^{-0.04\cdot\frac{1}{4}}\Phi(w - 0.3\sqrt{\frac{1}{4}})$

$w = \dfrac{rt + \frac{\sigma^2}{2}t - \log(\frac{K}{S})}{\sigma\sqrt{t}} = \dfrac{0.04\cdot\frac{1}{4} + \frac{0.3^2}{2}\cdot\frac{1}{4} - \log(\frac{100}{95})}{0.3\cdot\sqrt{\frac{1}{4}}}$

then $C = \cdots\cdots$

ⓑ $P(S(t) < 100) = P(S(0)e^{W} < 100)$

$= P(W < \log(\frac{100}{95}))$

$= \Phi\left(\dfrac{\log(\frac{100}{95}) - \frac{0.05}{4}}{0.3\sqrt{\frac{1}{4}}}\right)$

$= \Phi(0.2586) = 0.602.$

ⓒ $\begin{cases} \text{if } P(S(t) > 105)\cdot P(S(t) > S(0.5)) = P. & \text{the return } 50, \\ \text{otherwise} & 0 \end{cases}$

$\text{Return} = \begin{cases} 50e^{-r\cdot 1} - C & P \\ -C & 1-P \end{cases}$

$E[\ ] = 0 \Rightarrow P\cdot 50e^{-r\cdot 1} - CP - C + CP = 0$

$\Rightarrow C = P\cdot 50e^{-r\cdot 1}.$

thuse we need

$P(S(1) > 105)$ it is easy - —

$P(S(1) > S(0.5))$

7.7. $\text{Return} = \begin{cases} Fe^{-rt} - C & P(S(t) > K) \\ -C & \text{otherwise} \end{cases}$

$$E[\ ] = P\cdot(Fe^{-rt} - C) + (1-P)(-C) = 0$$

$$C = P \cdot Fe^{-rt}$$

$$= P(S(t) > K) \cdot 100\, e^{-0.03}$$

$$P(S(t) > 40) = P(S(0)e^{W} > 40) = P\left(W > \log\left(\frac{40}{38}\right)\right)$$

$$= \Phi\left(\frac{\log\left(\frac{40}{38}\right) - \left(0.06 - \frac{(0.32)^2}{2}\right)\cdot\frac{1}{2}}{0.32\cdot\sqrt{\frac{1}{2}}}\right)$$

$\left(r - \frac{\sigma^2}{2}\right)$.

7.8 $e^{-0.03} \cdot 100 \underbrace{P(S(t)>K)}$

$$\parallel$$

$$\Phi\left(\frac{\log\left(\frac{40}{38}\right)-0}{0.32\sqrt{\frac{1}{2}}}\right)$$

7.10. ⓐ Return $= \begin{cases} 5e^{-r\cdot 1}-10. & \text{if } S(1)<95 \quad P_1 \\ xe^{-r\cdot 1}-10 & \text{if } S(1)>110. \quad P_2 \\ 0. & \text{otherwise} \end{cases}$

$$E=0 \Rightarrow P_1(5e^{-r\cdot 1}-10)+P_2(xe^{-r\cdot 1}-10)=0.$$

$$(*) \Rightarrow xe^{-r}\cdot P_2 = 10P_2+10P_1-5P_1e^{-r}$$

$$P_1 = P(S(1)<95) = P\left(\log\frac{S(1)}{S(0)} < \log\left(\frac{95}{100}\right)\right)$$

$\mu = r-\frac{\sigma^2}{2} = -0.02$ $\quad = P\left(\frac{\log\frac{S(1)}{S(0)}-\mu t}{0.4} < \frac{\log\left(\frac{95}{100}\right)-(-0.02)}{0.4}\right)$

$\sigma = 0.4.$

risk-neutral $= \Phi\left(\frac{-0.0313}{0.4}\right) = 0.469$

$P_2 = 0.386$

so by $(*)$ we have $x = \ldots$

ⓑ $P(S(1)<95) = P\left(\frac{\log\left(\frac{S(1)}{S(0)}\right)-0.05\cdot 1}{0.4} < \frac{\log\left(\frac{95}{100}\right)-0.05}{0.4}\right)$

$= 1-\Phi(0.25325) = 40\%$

7.11

$$\text{Return} = \begin{cases} 100 e^{-r} - C & S(0.5) > 42 \text{ or } S(1) > 1.05\, S(0.5) \quad \underline{1-p} \\ -C & \text{otherwise} \quad : p \end{cases}$$

$$E = 0 \Rightarrow (1-p)(100e^{-r} - c) + (-c)p = 0$$

$$r - \frac{\sigma^2}{2} = 0.06 - \frac{0.16}{2} = \boxed{-0.02}$$

$$\Rightarrow (1-p)\, 100 e^{-r} = c.$$

$$p = P(S(0.5) < 42 \cap P(S(1) < 1.05\, S(0.5)))$$

$$= P(S(0.5) < 42) \cdot P(S(1) < 1.05\, S(0.5))$$

$$P(S(0.5) < 42) = P\left(\log \frac{S(0.5)}{S(0)} < \log \frac{42}{40}\right) = P\left(Z < \frac{\log\left(\frac{42}{40}\right) - (-0.02)\frac{1}{2}}{0.4 \cdot \sqrt{\frac{1}{2}}}\right)$$

$$= \Phi(0.2078) = 0.590$$

$$P(S(1) < 1.05\, S(0.5)) = P\left(\log \frac{S(1)}{S(0.5)} < \log 1.05\right) = P\left(Z < \frac{\log(1.05) - (-0.02) \cdot \frac{1}{2}}{0.4 \cdot \sqrt{\frac{1}{2}}}\right)$$

$$= \Phi(0.1720) = 0.569$$

so $P = 0.336$.

$$\Rightarrow 0.664 \cdot 100 \cdot e^{-0.06} = 62.53 = C$$

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(b) Under the actual drift parameter

$$P(S(0.5) < 42) = \Phi\left(\frac{\log\left(\frac{42}{40}\right) - 0.03}{\sqrt{0.08}}\right) = \Phi(0.664) = 0.527$$

$$P(S(1) < 1.05\, S(0.5)) = \Phi\left(\frac{\log(1.05) - 0.03}{\sqrt{0.08}}\right) = \Phi(-0.0280)$$
$$= 0.489$$

so $\quad P = 1 - 0.527 * 0.489 = 0.742$

7.12

$$\text{Return} = \begin{cases} 100e^{-r} - C & S(1) > (1+X)40 \\ -C & \end{cases}$$

$$r - \frac{\sigma^2}{2} = 0.02 - \frac{(0.2)^2}{2} = 0.$$

so $\quad C = P \cdot 100e^{-r}$

$$P = P(S(1) > (1+X)40) = P\left(\frac{\log \frac{S(1)}{S(0)}}{0.2} > \frac{\log(1+X)}{0.2}\right)$$

($S(0) = 40$)

$$= P\left(Z > \frac{\log 1+X}{0.2}\right) = 1 - \Phi\left(\frac{\log 1+X}{0.2}\right)$$

$$10 = 100e^{-0.02} \cdot p \Rightarrow p = 0.1020.$$

$$\Rightarrow \Phi\left(\frac{\log 1+X}{0.2}\right) = 0.8980 \Rightarrow \frac{\log 1+X}{0.2} = 1.27$$

$$\Rightarrow X = 0.2892$$

(b). $P\left(\frac{S(1)}{S(0)} > 1.2892\right) = P\left(\frac{\log\frac{S(1)}{S(0)} - 0.04}{0.2} > \frac{\log 1.2892 - 0.04}{0.2}\right)$

$= 1 - \Phi(1.070) = 0.157$