Chap 6.

6.1 $P_i = \frac{1}{1+O_i}$

$\Rightarrow P_1 = \frac{1}{2} \quad P_2 = \frac{1}{3} \quad P_3 = \frac{1}{6} \quad \sum_{i=1}^{3} P_i = 1$

thus. $(P_1\ P_2\ P_3)$ is the probability to make all bets fair

no arbitage is present.

6.2 $\frac{1}{1+2} + \frac{1}{1+3} + \frac{1}{1+4} + \frac{1}{1+O_4} = 1 \Rightarrow O_4 = \frac{47}{13}$

6.3 ⓐ $E[r_1] = P_1\, r_1(1) + P_2\, \cancel{r_1}(2) + P_3\, r_1(3) = P_1 4 + 8P_2 - 10P_3$

$E[r_2] = 6P_1 + 12P_2 - 16P_3$.

$P_3 = 1 - P_1 - P_2$

thus. $\begin{cases} 14P_1 + 18P_2 - 10 = 0 \\ 22P_1 + 28P_2 - 16 = 0 \end{cases} \Rightarrow \begin{matrix} P_1 = 2 \\ P_2 = -1 \end{matrix}$ not possible

so there is arbitrage.

ⓑ $6P_1 + (-3)P_2 + 0 \cdot P_3 = 0$

$-2P_1 + (0)P_2 + 6P_3 = 0$

$10P_1 + (10)P_2 + XP_3 = 0 \Rightarrow \boxed{X = -90.}$

$P_1 = 0.3$

$P_2 = 0.6.$

$P_3 = 0.1$

6.4 From 6.1. $P_1 = \frac{1}{2}$ $P_2 = \frac{1}{3}$ $P_3 = \frac{1}{6}$.

	outcome	odds	
wager $\leftarrow$	$(1,2)$	O_{12}	$E[\text{return}] = O_{12} \cdot (P_1 + P_2) - 1 \cdot P_3 = 0$
	(2.3)	O_{23}	$E[\text{return}] = O_{23} (P_2 + P_3) - 1 \cdot P_1 = 0$
	(1.3)	O_{13}.	$E[\text{return}] = O_{13} (P_1 + P_3) - 1 \cdot P_2 = 0$

$$\Rightarrow O_{12} = \frac{1}{5}$$
$$O_{13} = \frac{1}{2}$$
$$O_{23} = 1$$

6.5 betting scheme x_i. $i = 1 \dots m$.

If the outcome is j, then.

$$O_j x_j - \sum_{i \neq j} x_i = O_j x_j - \sum_{i=1}^{m} x_j + x_j$$

$$= (1 + O_j) x_j - \sum_{i=1}^{m} x_j$$

$$= \frac{(1+O_j) \cdot (1+O_j)^{-1}}{1 - \sum_{i=1}^{m} (1+O_i)^{-1}} - \frac{\sum_{i=1}^{m} (1+O_i)^{-1}}{1 - \sum (1+O_i)^{-1}}$$

$$= 1$$

6.6.

The present value to purchase one put option is

$$\text{return} = \begin{cases} -P & \text{if price is } 200. \\ \dfrac{100}{1+r} - P & \text{if price is } 50. \end{cases}$$

$$E[\text{return}] = p \cdot (-P) + (1-p)\left(\frac{100}{1+r} - P\right)$$

$$= p(-P) + (1-p)\frac{100}{1+r} + (1-p)(-P)$$

$$= -\cancel{pP} + (1-p)\frac{100}{1+r} - P + \cancel{pP}$$

$$\Rightarrow P = (1-p)\frac{100}{1+r} = \underbrace{\left(1 - \frac{1+2r}{3}\right)\frac{100}{1+r}}$$

$$S + P - C = \frac{K}{1+r} \Rightarrow C = S + P - \frac{K}{1+r}$$

6.7

100 → 200 → 400 @ $\frac{1}{3}\cdot\frac{1}{3}$

100 → 200 → 100 @ $\frac{1}{3}\cdot\frac{2}{3}\cdot 2$

100 → 50 → 100

100 → 50 → 25 @ $\frac{2}{3}\cdot\frac{2}{3}$

$$p = \frac{1+r-d}{u-d} = \frac{1-0.5}{2-0.5} = \frac{1}{3}$$

assuming $\underline{\underline{r=0}}$

own an option Return func is

$$\begin{cases} \dfrac{250}{(1+r)^2} - C & @ \frac{1}{9} \\ -C & @ \frac{4}{9} \\ -C & @ \frac{4}{9} \end{cases}$$

$$E[\text{return}] = \frac{1}{9}[250 - C] - \frac{4}{9}C - \frac{4}{9}C = 0$$

$$\Rightarrow \boxed{C = \frac{250}{9}}$$

6.8.

$$100 \begin{cases} 50 \\ 100 \\ 200. \end{cases} \qquad K=150$$

return of purchase stock. $= \begin{cases} \frac{200}{1+r} - 100 & P_1 \\ \frac{100}{1+r} - 100 & P_2 \\ \frac{50}{1+r} - 100. & P_3 \end{cases}$

$$E[\text{return}] = \left(\frac{200}{1+r} - 100\right) P_1 + \left(\frac{100}{1+r} - 100\right) P_2 + \left(\frac{50}{1+r} - 100\right) P_3$$
$$= 0$$

return of purchase an option $= \begin{cases} \frac{50}{1+r} - C & \text{if price is } 200 \\ -C & \text{if price is } 100 \\ -C & \text{if price is } 50. \end{cases}$

$$E[\text{return}] = P_1\left(\frac{50}{1+r} - C\right) + P_2(-C) + P_3(-C)$$
$$= P_1 \frac{50}{1+r} \cancel{+} - (P_1 + P_2 + P_3) C = 0$$

$$\Rightarrow \quad C = P_1 \cdot \frac{50}{1+r}$$

From $\left(\frac{200}{1+r}\right) P_1 + \left(\frac{100}{1+r}\right) P_2 + \left(\frac{50}{1+r}\right) P_3 = 100(P_1 + P_2 + P_3) = 100$

$$\Rightarrow \quad 2P_1 + P_2 + \frac{1}{2} P_3 = (1+r)$$
$$\Rightarrow \quad 2P_1 + P_2 + \frac{1}{2}(1 - P_1 - P_2) = 1+r$$
$$\Rightarrow \quad 2P_1 + P_2 - \frac{1}{2} P_1 - \frac{1}{2} P_2 + \frac{1}{2} = 1+r$$
$$\Rightarrow \quad \frac{3}{2} P_1 + \frac{1}{2} P_2 = \frac{1}{2} + r$$
$$\Rightarrow \quad 3P_1 + P_2 = 1 + 2r$$

$\cancel{\Leftarrow} \Rightarrow P_1 = \dfrac{1+2r-P_2}{3}$

$$C = P_1 \cdot \frac{50}{1+r} = \frac{1+2r-P_2}{3} \cdot \frac{50}{1+r}$$

$0 < P_2 < 1$

$$\Rightarrow \boxed{\frac{1+2r}{3} \cdot \frac{50}{1+r} > C > \frac{2r}{3} \cdot \frac{50}{1+r}}$$