§ 4.1

ⓐ $\left(1+\frac{10\%}{2}\right)^2 - 1 = 0.1025$

ⓑ $\left(1+\frac{10\%}{4}\right)^4 - 1 = 0.1038$

ⓒ $e^{0.1} - 1 = 0.1052$

4.2. $\cancel{P+}$ $pe^{0.1t} = 2p \Rightarrow t = \frac{\log 2}{0.1} = 6.93$

4.3. $(1+0.05)^t = 4 \Rightarrow t = 28$ yrs

$(1+0.04)^t = 4 \Rightarrow t = 35$ yrs.

4.4. $(1+r)^t = 3 \Rightarrow t = \frac{\log 3}{\log(1+r)}$

4.5 $A + A\left(1+\frac{0.06}{12}\right) + A\left(1+\frac{0.06}{12}\right)^2 + \cdots = 10000$

$$A \sum_{i=0}^{59} \left(1+\frac{0.06}{12}\right)^i = 10000$$

$$A \cdot \frac{1-1.005^{60}}{1-1.005} = 10000$$

$$A = 142.6.$$

4.6

$$PV = -1000 - \frac{1200}{1.06} + \frac{800}{1.06^2} + \frac{900}{1.06^3} + \frac{800}{1.06^4} = -30.75$$

that means "this cash flow is equivalent to at =0. one already lose 30.75 \$", it is not worthwhile.

4.7

$$S_1 = \frac{20}{1+r} + \frac{20}{(1+r)^2} + \frac{20}{(1+r)^3} + \frac{15}{(1+r)^4} + \frac{10}{(1+r)^5} + \frac{5}{(1+r)^6}$$

$$S_2 = \frac{10}{1+r} + \frac{10}{(1+r)^2} + \frac{15}{(1+r)^3} + \frac{20}{(1+r)^4} + \frac{20}{(1+r)^5} + \frac{20}{(1+r)^6}$$

① $r=0.03$ $S_1 = 82.71$ $S_2 = 84.63$ 2nd is better

② $r=0.05$ $S_1 = 78.37$ $S_2 = 78.60$ 2nd is better

③ $r=0.10$ $S_1 = 69.01$ $S_2 = 65.99$ 1st is better

4.8.

$$PV = -10000 + \frac{500}{(1+\frac{r}{2})} + \frac{500}{(1+\frac{r}{2})^2} + \cdots + \frac{500}{(1+\frac{r}{2})^{10}} + \frac{10000}{(1+\frac{r}{2})^{10}}$$

$\Rightarrow$ ① $r = 0.06$ $S = 1706$

$\Rightarrow$ ② $r = 0.1$ $S = 0$

$\Rightarrow$ ③ $r = 0.12$ $S = -736$

4.9.

$$4200 - 1000 = \frac{160}{(1+r)} + \frac{160}{(1+r)^2} + \cdots + \frac{160}{(1+r)^{24}}$$

$$\Rightarrow \frac{1 - \left(\frac{1}{1+r}\right)^{24}}{r} = 20. \Rightarrow r = 0.015$$

15% effective interest rate.

4.12 the bank charge 2 pts then we only recieve

$12{,}000 * 0.98 = 117{,}600$ \$

but we need pay interest $12000 * 0.05 = 600$

the cash flow is

0	1	2	....	35	36
117600	-600	-600		-600	-120000 -600

$$117600 = \frac{600}{(1+r)} + \frac{600}{(1+r)^2} + \dots + \frac{600}{(1+r)^{35}} + \frac{120600}{(1+r)^{36}}$$

$$= \frac{600\left[1 - \left(\frac{1}{1+r}\right)^{35}\right]}{r} + \frac{120600}{(1+r)^{36}}$$

$\Rightarrow r \approx 0.5615\%$.

4.13. cash flow I 16,000.

cash flow II $\quad 10{,}000, \quad 10{,}000 \quad \dots \quad 10{,}000.$

$\cancel{(1+r)^0} \quad \cancel{(1+r)^1} \quad \dots \quad \cancel{(1+r)^{10}}$

$e^{r \cdot 0} \quad e^{r1} \quad \dots \quad e^{r10}$

$PV_1 = 16000$

$PV_2 = 10000 + \frac{10000}{e^r} + \frac{10000}{e^{2r}} + \dots + \frac{10000}{e^{10r}}$

① $r = 0.02 \quad PV_2 = 18187$ so I is better

② $r = 0.05 \quad PV_2 = 16065$ I is better

③ $r = 0.10 \quad PV_2 = 13678$ II is better.

4.14.

$$-1000 \quad 30 \quad 30 \quad \cdots \quad 30 \quad 1030$$

$$t=0 \quad t=1 \quad \cdots \quad t=10$$

$$e^{r\cdot\frac{1}{2}}$$

$$-1000+\frac{30}{e^{\frac{0.05}{2}}}+\frac{30}{e^{0.05}}+\frac{30}{e^{0.05*\frac{3}{2}}}+\cdots+\frac{30}{e^{0.05*\frac{9}{2}}}+\frac{10\ldots}{e^{0.0\ldots}}$$

$$=\underline{40.94}$$

4.15. $\cancel{y=\left(1+\frac{0.05}{x}\right)^{x}} \quad \cancel{\ln y = x\ln\left(1+\frac{1}{20x}\right)}$

4.15. For Compound rate 0.05. $\left(1+\frac{0.05}{n}\right)^{n}$ is for deposit 1\$ for n time period.

4.16. after n days

$$100\left(e^{\frac{0.06}{365}\cdot n}-1\right)$$

4.17. $1000\cdot e^{3r}+2000e^{2r}+3000e^{r}$

4.18.

$$\frac{3}{(1+0.05)}+\frac{5}{(1+0.05)^2}-\frac{6}{(1+0.05)^3}+\frac{5}{(1+0.05)^4}=6.74.$$

4.19. $20+\frac{10}{(1+r)}>0+\frac{34}{(1+r)} \Rightarrow r>0.2$

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4.20. $104e^{-r} = 110e^{-2r}$

$$\Rightarrow \quad r = \log\frac{110}{104} = 0.0561$$

4.21. $Ae^{-rs} + Ae^{-r(s+t)} + Ae^{-r(s+2t)} + \cdots$

$$= Ae^{-rs}\sum_{n=0}^{\infty}\left(e^{-rt}\right)^n = \frac{Ae^{-rs}}{1-e^{-rt}}$$