Hw #1

1.1 (a) $1 - P_0 - P_1 - P_2 - P_3 = 0.05$

(b) $P_0 + P_1 + P_2 = 0.80.$

1.2. $P(\text{picnic not postponed}) = 1 - P(\text{postpone})$

$= 1 - P(\text{cloudy} \cup \text{raining})$

$= 1 - P(C \cup R)$

$P(C \cup R) = P(C) + P(R) - P(C \cap R)$

$= .4 + .3 - .2 = 0.5$

So. It is $1 - 0.5 = 0.5$

1.3. (a) $\frac{8}{14} \cdot \frac{7}{13}$

(b) $\frac{6}{14} \cdot \frac{5}{13}$

(c) $\frac{6}{14} \cdot \frac{8}{13} + \frac{8}{14} \cdot \frac{6}{13}$

1.4. (a). $\frac{27}{58}$

(b). $\frac{27}{35}$

1.6. Let A be the event that they are both aces
B the event that they are different suit

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{\frac{4}{52}\cdot\frac{3}{51}}{P(B)}$$

$$P(B) = 1 - P(\text{same suit})$$

$$= 1 - \frac{\binom{52}{1}\binom{12}{1}}{\binom{52}{1}\binom{51}{1}} = 1 - \frac{52\times 12}{52\times 51} = \frac{39}{51}$$

$$\Rightarrow P(A|B) = \frac{\frac{4}{52}\cdot\frac{3}{51}}{\frac{39}{51}} = \frac{1}{169}$$

1.8. (a) X : the win amount of the gamber

$$X = \begin{cases} 1 & 1-\left(\frac{20}{38}\right)^2 = \frac{100}{361} \\ -3 & \text{— lose at both times. } \left(\frac{20}{38}\right)^2 = \frac{261}{361} \end{cases}$$

$$p(X>0) = \frac{100}{361}$$

(b) $$E[X] = 1\cdot\frac{100}{361} - 3\cdot\frac{261}{361} = \frac{-39}{361}$$

1.10. (a) $P(N=2) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ (A win 2 or B win 2)

$P(N=3) = 1 - \frac{1}{2} = \frac{1}{2}$ (N can only be 2 or 3)

$$E[N] = 2\cdot\frac{1}{2} + 3\cdot\frac{1}{2} = 2.5$$

$$Var[N] = \frac{1}{2}(2-2.5)^2 + \frac{1}{2}(3-2.5)^2 = \frac{1}{4}$$

1.11 $\quad Var(x) = E[(x-E[x])^2]$

$= E[x^2 - 2E[x]x + E[x]^2)$

$= E[x^2] - 2E^2[x] + E[x]^2 = E[x^2] - (E[x])^2$

1.12. F be the fee if she takes the fixed amount

X is the event if she takes.

$E[F] = 5000 \quad Var[F] = 0 \quad (F \text{ is constant})$

$E[x] = 25000 * 0.3 + 0. * 0.7$

$= 7.500$

$E[x^2] = (25000)^2 * 0.3 + 0*(0.7) = 1.875 \times 10^8$

$Var(x) = E[x^2] - (E[x])^2 = 1.875 * 10^8 - (7500)^2$

$= 1.3125 * 10^8$

$\sqrt{Var(x)} = \sqrt{1.3125} * 10^4$

1.13. (a) (b) are obvious.

(c) $\sum_{i=1}^{n} (X_i - \bar{X})^2 = \sum_{i=1}^{n} X_i^2 - 2X_i\bar{X} + \bar{X}^2$

$= \sum_{i=1}^{n} X_i^2 - 2\bar{X}\sum X_i + n\bar{X}^2$

$= \sum_{i=1}^{n} X_i^2 - 2\bar{X}\, n\bar{X} + n\bar{X}$

$= \sum_{i=1}^{n} X_i^2 - n\bar{X}^2$

(d) $E[S^2] = \frac{1}{n-1}\left(\sum_{i=1}^{n} E[X_i^2] - n E[\bar{X}^2]\right)$

$= \frac{1}{n-1}\left(n \cdot E[X_1^2] - n E[\bar{X}^2]\right)$

$= \frac{1}{n-1}\left(n \cdot \left(Var(X_1) + E^2[X_1]\right) - n\left(Var(\bar{X}) + E^2[\bar{X}]\right)\right)$

$= \frac{1}{n-1}\left(n \cdot \left(\sigma^2 + \mu^2\right) - n\left(\frac{\sigma^2}{n} + \mu^2\right)\right)$

$= \frac{1}{n-1}\left((n-1)\sigma^2\right) = \sigma^2$

1.14

obvious

1.15. obvious.

1.16. $Cov(aU + bV, cU + dV)$

$= Cov(aU, cU + dV) + cov(bV, cU + dV)$

$= Cov(aU, cU) + cov(aU, dV)$

$+ Cov(bV, cU) + Cov(bV, dV)$

$= ac \cdot Var(U) + ad\, Cov(U, V) + bc\, Cov(V, U) + bd\, Var(V)$

$= ac + bd$

1.17 (a) $Cov(X_1+X_2, X_3+X_4) = Cov(X_1, X_3) + Cov(X_1, X_4) + Cov(X_2, X_3) + Cov(X_2, X_4)$

$= 3 + 4 + 6 + 8 = 21$

(b) Similar

$X_1 = \begin{cases} 1 @ \frac{1}{2} \\ -1 @ \frac{1}{2} \end{cases}$ $E(x) = 0$.

$Var(X_1) = \frac{1}{4}$

1.18. $Y = X_1 + X_2 + X_3$.

$Cov(X_1, X_1+X_2+X_3) = Cov(X_1, X_1) = Var(X_1) = \frac{1}{4}$.

$Corr(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X) Var(Y)}} = \frac{\frac{1}{4}}{\sqrt{\frac{1}{4} \cdot \frac{3}{4}}} = \frac{1}{\sqrt{3}}$.