# Math708 - Final Project

1. (25 points) Prove or disprove: if $f \in C([-1,1])$ has mean value zero, then a best approximation to $f$ by polynomials of degree at most $n$ (minimax) with mean value zero is a best approximation to $f$ among all polynomials of degree $n$.

2. (25 points) Find the simple quadrature rule of highest degree of precision for estimating $\int_{-1}^{1} f(x)dx$ in terms of the values of $f$ at $-1/2, 0$, and $1/2$. Give a complete convergence analysis for the corresponding composite quadrature rule.

3. (25 points) For the initial-value problem $y' = f(x,y)$ with $y(x_0) = y_0$, show that the one-step method defined by

$$y_{n+1} = y_n + \frac{1}{2}h(k_1 + k_2)$$

   where

$$k_1 = f(x_n, y_n), k_2 = f(x_n + h, y_n + hk_1)$$

   is consistent and find the truncation error.

4. (25 points) For the initial-value problem $y' = f(x,y)$ with $y(x_0) = y_0$, determine the order of the linear multistep method

$$y_{n+2} - (1+a)y_{n+1} + y_n = \frac{1}{4}h[(3-a)f_{n+2} + (1-3a)f_n]$$

   and investigate its zero-stability and absolute stability.

5. (25 points) For the initial-value problem $y' = f(x,y)$ with $y(x_0) = y_0$, consider the $\theta$-method

$$y_{n+1} = y_n + h[(1-\theta)f_n + \theta f_{n+1}]$$

   for $\theta \in [0,1]$. Show that the method is A-stable if, and only if, $\theta \geq 1/2$.