

Math708 - Homework 3

1. Construct orthogonal polynomials of degree 0, 1 and 2 on the interval $(0, 1)$ with the weight function $w(x) = -\ln x$.
2. The Newton-Cotes formula with $n = 3$ on the interval $[-1, 1]$ is $\int_{-1}^1 f(x)dx \approx w_0f(-1) + w_1f(-1/3) + w_2f(1/3) + w_3f(1)$. Find the values of the weights w_0, w_1, w_2 and w_3 , and give the error estimate.
3. (Computer Exercise) Using n equally spaced nodes on the interval $[-5, 5]$, find the interpolating linear spline and natural cubic spline for the function $f(x) = (x^2 + 1)^{-1}$. Plot two functions with different values of n ($n = 5, 11, 21$), and observe the discrepancy between $f(x)$ and $p(x)$.
4. (Computer Exercise) Apply composite *midpoint*, *trapezium* and *Simpson* rules to evaluate

$$\int_0^{2\pi} \cos(2x)e^{-x} dx.$$

with error tolerance $\epsilon = 1.0 \times 10^{-5}$.

5. (Computer Exercise, Extra Credits) Apply Fast Fourier Transform (FFT) to solve

$$\int_{-\pi}^{\pi} \cos(x)e^{x^2} dx.$$

with $n = 1000$ equally spaced points using *left end point* rule.