Math708 - Homework 2

1. Prove or disprove: if $f \in C([-1, 1])$ is odd (even), then a best approximation (with ∞ norm) to f by odd (even) polynomials of degree at most n is a best approximation to f among all polynomials of degree n.

2. Construct the minimax polynomial $p_2 \in P_2$ on the interval $[-1, 1]$ for the function $f(x) = \sin x$.

3. (Computer Exercise) Use orthogonal polynomials, find the quadratic polynomial that fits the following data in the sense of lease squares:

   a:

x	-1	-0.5	0	0.5	1
y	-1	0	1	2	1

   b:

x	-2	-1	0	1	2
y	2	1	1	1	2

4. (Computer Exercise, extra credits) Use Remez algorithm to find the minimax polynomial $p_5 \in P_5$ on the interval $[-1, 1]$ for the function $f(x) = \sin x$.