New Functions from Old

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Overview

In this lab, we will use Maple to help us to calculate and simplify combinations of functions. A maplet (**Shift)** will also be introduced to help us to practice our skills identifying basic functions that have been shifted horizontally and/or vertically.

Maple Essentials

- Important Maple commands introduced in this lab:

Command	Description	Example
`simplify`	simplify functions/expressions	`simplify(f(x));` `simplify(x-(x-1)^8);`
`eval`	evaluate functions/expressions	`eval(f(x),x=2);` `eval((x-h)^9,h=0);`

- The **Shift** maplet is available from the course website:

  `http://www.math.sc.edu/calclab/141L-F07/labs/` → Shift

Related course material/Preparation

§1.3 *New Functions From Old* (Pages 27-39) of the textbook (Anton, 8th edition).

Assignment

With help of Maple, work out Exercises 11–14, 30, 32, 40, and 56 of §1.3 (pages 36–38).

Activities

1. Use the **Shift** maplet to practice your skills identifying basic functions that have been shifted horizontally and/or vertically.
   (a) From the Calculus I Lab Schedule/Assignments page under Lab 3, click on Shift. You will be prompted for a username and password as these maplets are protected. You should use your Blackboard username and password. (You may need to reset your password there: login to your VIP, go to the TECHNOLOGY, choose the second one from the TECHNOLOGY Menu.)
   (b) This opens a user interface for testing your ability to recognize shifts of seven basic functions. To see the seven basic functions, click the **Show Basic 7 Functions** button.
   (c) To test your ability to recognize shifts of these functions, click on the **Show Shifted Graph** button. Enter the formula for the displayed graph (using valid Maple syntax) in the box labeled *Answer*, then click the **Check Answer** button.

   Note: *If you do not get the answer correct, the graph of your equation will be displayed in red.*

2. In each of the following problems, you will use the assignment operator (`:=`) together with the arrow notation (`x->`) to define each function. Once you have done this, the problems are straightforward. Remember that you have to call a function **together with its variable** (like f(x), f(t), f(2), or f(whatever)). Maple won't recognize a function just by his name (like f).
   Note: *You can always use the Expression, Common Symbols, and/or Favorites palettes to avoid typing so much. You may also find the labels useful.*
   - Find and simplify formulas for $f(x)+g(x)$, $f(x)-g(x)$, $f(x)g(x)$, and $f(x)/g(x)$.
     a. $f(x)=2\sqrt{x-1},\ g(x)=\sqrt{x-1}$ (Ex. 29 on P.37)
     b. $f(x)=1+\frac{x}{x+1},\ g(x)=2-\frac{1}{x}$
   - Let $f(x)=x^2+1$ (Ex. 33 on P.37). Find and simplify each of the following.
     a. $f(5s+2)$ b. $3f(1/x)$ c. $f(f(x))$
   - Evaluate $\frac{f(x+h)-f(x)}{h}$. Simplify your answer and then let h go to 0.
     a. $f(x)=3x^2-5$ (Ex. 53 on P. 38) b. $f(x)=\frac{1}{(x+1)^2}$
   - Find and simplify compositions $(f\circ g)(x)=f(g(x))$ and $(g\circ f)(x)=g(f(x))$.
     a. $f(x)=\frac{1+x}{1-x},\ g(x)=\frac{x}{1-x}$ (Ex. 37 on P. 37)
     b. $f(x)=\sqrt{2x+3},\ g(x)=x^2+1$
   - Find and simplify $(f\circ g\circ h)(x)$.
     a. $f(x)=\sqrt{x-1},\ g(x)=x^2+2,\ h(x)=\cos x$
     b. $f(x)=x^2+1,\ g(x)=\frac{1}{x},\ h(x)=x^3$ (Ex. 39 on P. 37)
   - Express $F(x)$ as a composition of two functions; that is, find $f(x)$ and $g(x)$ such that $F(x)=f\circ g(x)$. Use Maple to verify the composition (Ex. 43 on P. 37).
     Note: *Do not choose the identity ($y=x$) as one of your functions.*
     a. $F(x)=\sin^2 x$ b. $F(x)=\frac{3}{5+\cos x}$ c. $F(x)=(x^2+1)^{10}$
   - Express $F(x)$ as a composition of three functions; that is, find $f(x)$, $g(x)$, and $h(x)$ such that $F=f\circ g\circ h(x)$. Use Maple to verify the composition (Ex. 45 on P. 37).
     Note: *Do not choose the identity ($y=x$) as one of your functions.*
     a. $F(x)=(1+\sin(x^2))^3$ b. $F(x)=\sqrt{1-x^{1/3}}$ c. $F(x)=\cos^4(\sqrt{x})$

Example Problems

1. Evaluate the difference quotient $\frac{f(x+h)-f(x)}{h},\ h\neq 0$ if $f(x)=\frac{4}{3+x^2}$. Simplify your answer and then let h go to 0.
   ```
   > f:= x -> 4 / (3+x^2);
   > (f(x+h) - f(x)) / h;
   > simplify(label);
   > eval(label, [h = 0, x = x]);
   ```
   Note: *You should right-click over your expression and choose an action. If you choose to type the command, use* **Ctrl-L** *to insert a label.*
2. Find and simplify $f\circ g\circ h(x)$ if $f(x)=\frac{2}{1-x^2}$, $g(x)=\sin(x)$, and $h(x)=\sqrt{x}$.
   ```
   > f:= x -> 2 / (1-x^2);
   > g:= x -> sin(x);
   > h:= x -> sqrt(x);
   > f(g(h(x)));
   > simplify(label);
   ```