Name _____

Directions: Answer each question on separate sheets of paper. Please clearly number each problem and circle your final answer. To receive full credit, you must show all work. The use of calculators is not allowed. There is a total of 100 points in the test.

1. (10 points) Calculate the sum of the series: $\displaystyle\sum_{k=0}^{\infty} 10\frac{2^{k+1}}{7^k}$

2. (15 points each) For each series, determine whether the series absolutely converges, conditionally converges or diverges. (Justify your answer, i.e. clearly state which test(s) you used and its results)

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^3+2}$

   (b) $\displaystyle\sum_{k=2}^{\infty} (-1)^{k+1}\frac{\ln k}{k}$

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-10)^k}{k^k}$

3. (15 points) Find the interval of convergence for the power series: $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{\sqrt{k}}x^k$

4. (15 points) Find the 3rd Taylor polynomial for $f(x) = \sin^2 x$ around the point $x = \frac{\pi}{4}$.

5. (15 points) Determine the Maclaurin series for $f(x) = \dfrac{3}{1-4x^2}$. What is its interval of convergence?

Extra-Credit (10 points) Using Maclaurin polynomials, estimate cos(1) accurately up to 6 decimal places. You should leave your answer as a sum of fractions using factorials, and give proof that your estimate is inside the margin of error.

n	$n!$
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800
12	479001600
13	6227020800
14	87178291200
15	$\approx 1.3 \times 10^{12}$
16	$\approx 2.1 \times 10^{13}$
17	$\approx 3.6 \times 10^{14}$
18	$\approx 6.4 \times 10^{15}$
19	$\approx 1.2 \times 10^{17}$
20	$\approx 2.4 \times 10^{18}$

n	$n!$
21	$\approx 5.1 \times 10^{19}$
22	$\approx 1.1 \times 10^{21}$
23	$\approx 2.6 \times 10^{22}$
24	$\approx 6.2 \times 10^{23}$
25	$\approx 1.6 \times 10^{25}$
26	$\approx 4.0 \times 10^{26}$
27	$\approx 1.1 \times 10^{28}$
28	$\approx 3.0 \times 10^{29}$
29	$\approx 8.8 \times 10^{30}$
30	$\approx 2.7 \times 10^{32}$
31	$\approx 8.2 \times 10^{33}$
32	$\approx 2.6 \times 10^{35}$
33	$\approx 8.7 \times 10^{36}$
34	$\approx 3.0 \times 10^{38}$
35	$\approx 1.0 \times 10^{40}$
36	$\approx 3.7 \times 10^{41}$
37	$\approx 1.4 \times 10^{43}$
38	$\approx 5.2 \times 10^{44}$
39	$\approx 2.0 \times 10^{46}$
40	$\approx 8.2 \times 10^{47}$