Answers to WS 7

1. $(36, 38)$.

Note that the equations are given as recursive rather than difference equations. To solve for equilibrium set $u_{n-1} = u_n = u$ and $v_{n-1} = v_n = v$.

2. a.

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{100 - N_1}{100} \right)$$
$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{150 - N_2}{150} \right)$$

b.

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{100 - N_1 - 2N_2}{100} \right)$$
$$\frac{dN_2}{dt} = r_2 N_2 \left(\frac{150 - N_2 - 3N_1}{150} \right)$$

c. The equilibrium values are $(0, 0)$, $(100, 0)$, $(0, 150)$, and $(40, 30)$.

d. In the short run species 1 decreases and species 2 increases. In the long run, species 1 becomes extinct, species 2 approaches equilibrium at its carrying capacity $K_2 = 150$.

3. The equilibrium values are $(0, 0)$, $(100, 0)$, and $(0, 150)$. There is no equilibrium value with both populations present. The two isoclines do not intersect. The isocline for species 1 is above the isocline for species 2.

a. in the short run both species decrease.

b. in the short run population 1 increases and population 2 decreases.

c. in the short run, both populations increase.

Long term outcome: species 2 becomes extinct, species 1 approaches its carrying capacity $K_1 = 100$. This is the outcome regardless of where the initial value point is.

4. The equilibrium values are $(0, 0)$, $(100, 0)$, $(0, 150)$, $(40, 120)$.

In the short run, both populations increase. The long term outcome: the two population coexist at the equilibrium value $(40, 120)$ (stable equilibrium), because the arrows point towards this point no matter where the initial value is chosen.