Answers to WS 6

1. u is not an eigenvector

v is an eigenvector with corresponding eigenvalue -3.

2. a. $B_1 = 6(1.4)v_1 - (0.8)v_2$, $B_2 = 6(1.4)^2 v_1 - (0.8)^2 v_2$.

b. $B_n = 6(1.4)^n v_1 - (0.8)^n v_2$

c.

$$\begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$$

c. in the long run, $P_n = 192(1.4)^n$. this indicates exponential growth with per capita growth rate $r = 40\%$

3.

a. 14%; 7.7%

b. 38.5%; 21.2%

c. 30.2%; 16.7% (note: the last part of the question cannot be answered based on the values in A^3; you can disregard the part concerning the probability for an adult to survive three consecutive years)

d.

$$A = \begin{bmatrix} 0 & 0 & 40 \\ 0.2 & 0 & 0 \\ 0 & 0.7 & 0.55 \end{bmatrix}$$

$$B_{20} = \begin{bmatrix} 222374086 \\ 21509873 \\ 10559716 \end{bmatrix} \quad D_{20} = \begin{bmatrix} 0.986 \\ 0.009 \\ 0.005 \end{bmatrix}$$

e. $P_8 = 75312$, $P_9 = 116336$, $P_10 = 278373$ We have $P_10 = 2.3928 P_9$ and $P_9 = 1.5447 P_8$. Since we need to multiply by a different number to get from P_n to P_{n+1} at $n = 8$ and at $n = 9$ we conclude that the exponential behavior has not been reached yet at $n = 8$.

It might be reached later but I will give full credit for an answer similar to the above (even if one could see exponential behavior if one goes further than $n = 10$). In other words you don't have to worry about how far you need to go. If you show that exponential behavior is not reached at the time step you chose, that is as good as showing that exponential behavior is reached later - you won't have to do both on the exam.