

**Math 172   Spring 2011   Worksheet 6**

1. We are given the following matrix and vectors:

$$A = \begin{bmatrix} -2 & 4 \\ 1 & 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Is  $u$  an eigenvector for the matrix  $A$ ? Justify your answer. If yes, what is the corresponding eigenvalue?

Is  $v$  an eigenvector for the matrix  $A$ ? Justify your answer. If yes, what is the corresponding eigenvalue?

2. A population is divided into two age classes and the transition matrix  $A$  has eigenvalues  $\lambda_1 = 1.4$  and  $\lambda_2 = 0.8$ . The corresponding eigenvectors are  $v_1$  and  $v_2$  and the initial population vector is  $B(0) = 6v_1 - v_2$ .

- a. Express  $B(1)$  and  $B_2$  in terms of  $v_1$  and  $v_2$ .
- b. Express  $B_n$  in terms of  $v_1$  and  $v_2$ .
- c. We are given

$$v_1 = \begin{bmatrix} 12 \\ 20 \end{bmatrix} \quad v_2 = \begin{bmatrix} 4 \\ 15 \end{bmatrix}$$

Use this information to find the stable distribution vector that is eventually reached when the population reaches a stable state.

- d. Describe the exponential behavior of the total population in the long run.

3. A frog population has three stages: tadpoles  $T_n$ , juveniles  $J_n$  and adults  $A_n$ .

Each year, 20% of tadpoles become juveniles and 80% of tadpoles die. There are no tadpoles that remain in the same stage at the next step. Also, 70% of juveniles become adults and 30% of juveniles die. There are no juveniles that remain in the same stage. 55% of adults survive, the rest die.

On average each adult produces 40 tadpoles a year. The tadpoles and juveniles don't reproduce.

- a. What is the probability for a tadpole to survive for two consecutive years (until it becomes adult)? Three consecutive years?
- b. What is the probability for a juvenile to survive for two consecutive years? Three consecutive years?
- c. What is the probability for an adult to survive for two consecutive years? Three consecutive years?
- d. Write the transition matrix and compute the population vector and distribution vector at  $t = 20$  if the initial population consists of 100 tadpoles, 20 juveniles, and 20 adults.

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e. Does the population have exponential behavior in the long run? Present numerical evidence for your answer and state what is the per capita growth rate  $r$ .