Math 172 Spring 2012 Worksheet 5

1. We are given the following matrices:

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 6 \\ -3 & 7 \end{bmatrix} \qquad C = \begin{bmatrix} 9 \\ 10 \end{bmatrix}$$

For each of the following operations calculate the resulting matrix or state that the operation is not possible:

$A \cdot B,\quad B \cdot A,\quad A \cdot C,\ \ C \cdot B,\ \ A^2$

2. A population consists of three age categories: children C_n, mature individuals M_n, and seniors S_n. The population vector B_n is

$$B_n = \begin{bmatrix} C_n \\ M_n \\ S_n \end{bmatrix}$$

The process described below takes place during each step:

25% of the children become mature individuals; 3% of children die

45% of the mature individuals become seniors; 8% of the mature individuals die; each pair of mature individuals produces two children

30% of the seniors die.

a. Set up a transition matrix that describes this process. Write down the recursive equation that shows how to get B_{n+1} from B_n using the transition matrix. Write down the formula for how to get B_n from the initial population vector B_0 using the transition matrix.

b. An initial population vector is given below:

$$B_0 = \begin{bmatrix} 50 \\ 100 \\ 200 \end{bmatrix}$$

Find the population vectors at $n = 3, n = 4$ and also at $n = 20, n = 21$.

c. Find the distribution vectors at $n = 3, n = 4, n = 20, n = 21$. Has a stable state been reached at $n = 3$? How about at $n = 20$?

d. Does the total population have an exponential behavior in the long run? Give numerical evidence to support your conclusion. If you answered yes, what is the value of the per captita growth rate r?