

Math 172 Spring 11 WS 4

1. An algal population grows in a lake and it is governed by the continuous model equation

$$\frac{dA}{dt} = 0.0003A(200 - A)$$

The initial population is $A(0) = 50$ tons.

a. Rewrite the equation in the form

$$\frac{dA}{dt} = rA \left(1 - \frac{A}{K} \right)$$

What are the numerical values of r and K ?

b. Apply stepwise estimation using Euler's method to estimate the population after 12 years. Use three steps, so the step size is $\Delta t = 4$.

c. Repeat b. but this time use 12 steps, so $\Delta t = 1$. Which of the two approximation is closer to the actual value?

d. Find the equilibrium value of this population and decide whether it is stable or unstable.

2. The growth rate of a population $P = P(t)$ is governed by the model equation

$$\frac{dP}{dt} = 0.04P \left(1 - \frac{P}{80} \right).$$

I. Determine all the equilibrium values.

II. Sketch the graph of $P(t)$ given each of the initial conditions:

a. $P(0) = 20$. Label this graph A.

b. $P(0) = 100$. Label this graph B.

c. $P(0) = 60$. Label this graph C.

d. Now suppose 0.04 is replaced by 0.01 in the model equation. Take $P(0) = 20$ again. Sketch the new graph and label it D.

For each of a.–d. above, explain why the graph has the shape that it does.

3. The growth of a population $N = N(t)$ is governed by the model equation

$$\frac{dN}{dt} = 0.04N \left(1 - \frac{N}{150} \right) \left(\frac{N}{30} - 1 \right)$$

I. Find all the equilibrium values and for each equilibrium value decide whether it is stable or unstable.

II. If $N(0) = 25$, sketch the graph of $N(t)$, and explain why the graph has the shape that it does.

III. Repeat II. with $N(0) = 50$.

4. The habitat where a population of fish lives is declining in quality, so that the per capita growth rate of the fish population is given by $r(t) = -0.8t + 3$, where t is measured in years.

The fish population is governed by the differential equation

$$\frac{dP}{dt} = r(t)P.$$

a. In the beginning (at $t = 0$) is the fish population increasing or decreasing?

b. After how many years does the fish population start to decrease?

c. Use the method of separation of variables to solve the differential equation.

d. What can you say about the long-term behavior of the fish population (becoming extinct, increasing without bound, or approaching a certain equilibrium value)?

5. A population of salamanders is down to 100 individuals, when the Nature Conservancy takes over their habitat and begins environmental remediation. The per capita growth rate of the salamander population, now at $r = -0.02$, is assumed to increase linearly in such a way that at $t = 20$ we have $r(t) = 0$, and thereafter $r(t)$ becomes positive.

a. Find the formula for $r(t)$.

b. Write the differential equation modeling the salamander population as a function of time.

c. Use the method of separation of variables to solve the equation in part b.

d. What happens to the salamander population in the long run?