Math 172 Spring Fall 2012 Worksheet 3

1. A doctor determines that her patient needs the drug digoxin and prescribes a dose of 1 mg per day. However, the kidneys remove one third of the digoxin present in the patient's blood each day. Let $N(t)$ denote the amount of digoxin present in the blood after t days.

a. Write a differential equation for $N(t)$.

b. Use the method of separation of variables in order to find the formula for $N(t)$.

c. Graph the function that you found in part b. Use your graph to predict the first time when the amount of digoxin in the patient's bloodstream will be above 2 mg.

d. Explain how the graph from c. reflects the type of the equilibrium value of $N(t)$ (stable or unstable).

2. A population $P(t)$ grows at a rate of 2% per year. Simultaneously, there is emigration of 12 million individuals per year.

a. Write the differential equation for this process.

b. Find the equilibrium value and decide whether it is stable or unstable.

c. Assume that the initial value is 700 million. Find the formula for $P(t)$ using the method of separation of variables.

d. Use the graph to decide if the population will ever reach 800 million. If so, how long does it take?

3. Due to poisoning, a mouse population is declining continuously at a rate of 5% per month. But food supplies are ample, so as the population declines, new immigrants move in steadily at a rate of 8 individuals per month.

a. Write a continous model equation to describe this process.

b. Is there an equilibrium value in which immigration is exactly balanced by the decline of the population? Find the equilibrium value and decide whether it is stable or unstable.

c. What happens if the population starts below the equilibrium value? What happens if the population starts above the equilibrium value?

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4. Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference between its temperature and the temperature of its surroundings.

a. Using T for temperature, t for time, k for the constant of proportionality, and T_s for the surrounding temperature (which is a constant), write a differential equation which models the object's temperature.

b. A fresh cup of coffee has a temperature of 90^o C and it is brought into a room where the surrounding temperature is 20^o C. Suppose that $k = -0.1$ (^{o}C per minute per ^{o} C of temperature difference).

Write the differential equation which models the coffee's temperature.

c. Solve the equation from part b.

d. Use your answer from c. to find the temperature of the cup of coffee after 10 minutes.

e. Use your answer from c. to estimate how long it takes for the coffee to reach 55 ^oC.

5. The habitat where a population of fish lives is declining in quality, so the per capita growth rate is given by $r(t) = -0.8t+3$ (t is measured in years). The differential equation is

$$\frac{dP}{dt} = r(t)P$$

a. In the beginning ($t = 0$) is the population increasing or decreasing?

b. After how many years does the fish population start to decrease?

c. Use the method of separation of variables to find $P(t)$.

d. Based on the formula found above, what can you say about the long term outcome of this population?

6. A population of salamanders is down to 100 individuals, when the Nature Conservancy begins environmental remediation of their habitat. The per capita growth rate of the population is now (at $t = 0$) at $r = -0.02$ and it is assumed to increase linearly in such a way that at $t = 40$ we will have $r(40) = 0$ (and thereafter r becomes positive).

a. Find the formula for the linear function $r(t)$.

b. The differential equation for the salamander population is

$$\frac{dP}{dt} = r(t)P$$

Use the method of separation of variables to solve this equation.

c. What happens to the population in the long run?