Math 172 Spring 2011 WS3

1. Due to poisoning, a mouse population is declining continuously at a rate of 5% per month. But food supplies are ample, so as the population declines, new immigrants move in steadily at a rate of 8 individuals per month.

a. Write a continous model equation to describe this process.

b. Is there an equilibrium value in which immigration is exactly balanced by the decline of the population? Find the equilibrium value and decide whether it is stable or unstable.

c. What happens if the population starts below the equilibrium value? What happens if the population starts above the equilibrium value?

2. An initial dose of 50 mg of quinine is given to a patient one time only. Quinine leaves the bloodstream at a rate of 6% per hour.

a. Write a differential equation for this process.

b. Write the explicit solution and determine how much quinine is left in the bloodstream after 12 hours.

c. At what time does just 5 mg of quinine remain?

3. A population grows at a rate of 2% per year. Simultaneously, there is emigration of 12 million individuals per year.

a. Write the model equation for this process.

b. Find the equilibrium value and decide whether it is stable or unstable.

c. Predict what will happen in the long run if the initial value is $A_0 = 200$ million.

d. Repeat part c. if $A_0 = 300$ million.

4. Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference between its temperature and the temperature of its surroundings.

a. Using T for temperature, t for time, k for the constant of proportionality, and T_s for the surrounding temperature (which is a constant), write a differential equation which models the object's temperature.

b. A fresh cup of coffee has a temperature of 90^o C and it is brought into a room where the surrounding temperature is 20^o C. Suppose that $k = -0.1$ (^{o}C per minute per ^{o} C of temperature difference).

Write the differential equation which models the coffee's temperature.

c. Solve the equation from part b.

d. Use your answer from c. to find the temperature of the cup of coffee after 10 minutes.

e. Use your answer from c. to estimate how long it takes for the coffee to reach 55 ^{o}C.

5. The differential equation

$$\frac{dP}{dt} = -.5P + 8$$

models a population of fish.

a. Find the equilibrium value and decide whetehr it's stable or not.

b. Assume that the initial population is $P(0) = 20$. Use either one of the two methods discussed in class to find the formula for $P(t)$.

c. What is the long term behavior of this population?

d. Repeat parts b. and c. if $P(0) = 10$.