Math 172 Fall 2012 WS 10 Answers

1. The equation for dP/dt is

$$\frac{dP}{dt} = -qP + \beta VP$$

The equation for dV/dt:

a.

$$\frac{dV}{dt} = rV - \alpha VP$$

b.

$$\frac{dV}{dt} = rV - \frac{kV}{D+V}P$$

c.

$$\frac{dV}{dt} = rV(1 - \frac{V}{K}) - \alpha VP = rV - cV^2 - \alpha VP$$

d.

$$\frac{dV}{dt} = rV(1 - \frac{V}{K}) - \frac{kV}{D+V}P = rV - cV^2 - \frac{kV}{D+V}P$$

2. a. Logistic behavior with carrying capacity of 500, because when $P =)$ the equation becomes

$$\frac{dV}{dt} = 10V - 0.02V^2 = 10V(1 - \frac{0.02}{10}V) = 10V(1 - \frac{V}{500})$$

b. $(0, 0)$, $(500, 0)$.

c. For the equation of dP/dt to be zero, we need $V = 40$. Now plug this into the equation of dV/dt. After V is factored out, this equation becomes

$$10 - 0.02V - \frac{1.2}{20+V}P = 0;$$

when we plug in $V = 40$ we get $9.2 - 0.02P = 0$ and solving for P gives $P = 9.2/0.02 = 460$. Thus the equilibrium pair is $(40, 460)$.

d. The isocline for P has equation $V = 40$. This is a vertical line.

The isocline for V has equation

$$10 - 0.02V - \frac{1.2}{20+V}P = 0$$

which we can write as

$$P = \frac{(10 - 0.02V)(20 + V)}{1.2} \tag{1}$$

This is the equation of an upside-down parabola.

e. By plugging in $V = 200$ in the equation (1) of the isocline above, we find $P = 1100$; by plugging in $V = 400$ we find $P = 700$.

f. $R(V) = \dfrac{1.2V}{20+V}$

g. 1.2

h. 20

3. a. exponential growth at a per-capita rate of 1000

b. $(0, 0)$.

c. The equation of P implies $V = 40$. Factor out V from the equation of V, then plug in $V = 40$:

$$10 - \frac{1.2V}{400 + V^2}P = 0$$

when we plug in $V = 40$ the equation becomes $10 - 0.024P = 0$, so $P = 10/0.024 = 416.7$. The equilibrium pair is $(40, 416.7)$

d. The isocline for P has equation $V = 40$; this is a vertical line. The isocline for V has equation

$$10 - \frac{1.2V}{400 + V^2}P = 0$$

which we can write as $4000+10V^2-1.2VP$; in order to graph this on the graphing calculator you can solve for P in terms of V: $P = (4000 + 10V^2)/1.2V$ and graph this as a function by using x for V and y for P).

e. when $V = 200$, $P = 1683$. When $V = 400$, $P = 3342$.

f. $R(V) = \dfrac{1.2V^2}{400 + V^2}$

g. 1.2

h. 20