1. Write the differential equation for a population that has logistic growth with a carrying capacity of 200 individuals, and an intrinsic growth rate of 3% per year.

2. Consider the logistic equation

$$\frac{dP}{dt} = P\left(1 - \frac{P}{200}\right)$$

a. Graph $P = P(t)$ if $P(0) = 10$. What is the value of $P(t)$ when $P(t)$ is increasing the fastest?

b. Graph $P = P(t)$ if $P(0) = 120$.

c. Graph $P = P(t)$ if $P(0) = 400$.

3. Consider the following variation of the logistic equation:

$$\frac{dP}{dt} = P\left(\frac{P}{50} - 1\right)\left(1 - \frac{P}{200}\right)$$

a. What is the long term outcome for $P(t)$ if $P(0) = 20$?

b. What is the long term outcome for $P(t)$ if $P(0) = 100$?

c. Graph $P = P(t)$ if $P(0) = 20$.

d. Graph $P = P(t)$ if $P(0) = 100$.

4. Consider the equation given in **2** with $P(0) = 50$. Use Euler's method with $\Delta t = 0.2$ to find the approximate value of $P(4)$ (note: you need to pay attention to how many steps are necessary in the Euler's method in order to reach $t = 4$).

5. A population with age structure has a transition matrix A. The matrix A has eigenvalues $\lambda_1 = 1.05, \lambda_2 = -0.2, \lambda_3 = 0.7$, with corresponding eigenvectors v_1, v_2, v_3.

a. Give the formula for the population vector at time n, u_n, in terms of the matrix A and the initial population vector u_0.

b. Assume that the initial population vector is $u_0 = 2v_1 + v_2 + 5v_3$. Give a formula for u_n in terms of v_1, v_2, v_3.

c. Assume that

$$v_1 = \begin{pmatrix} 8 \\ 1 \\ 7 \end{pmatrix}, v_2 = \begin{pmatrix} 5 \\ 2 \\ 5 \end{pmatrix}, v_3 = \begin{pmatrix} 4 \\ 12 \\ 3 \end{pmatrix}$$

Use your answer from b. to find the stable state distribution that this population will have in the long run.

d. Use the vectors v_1, v_2, v_3 given in part c. and your answer to part b. to decide whether the total population $P(n)$ has exponential behavior in the long run, and with what per capita growth rate.