

Math 172 Spring 2011 Project for the Honors Section students

Description: In this project you will solve more complicated differential equations (namely the logistic equation and the modified logistic equation with Alee effect) through separation of variables and integrating.

This project is due by the end of the semester. You should hand it complete solutions showing all the steps of the computations. Feel free to stop by my office if you have any questions.

Problem 1 Solve the differential equation

$$\frac{dP}{dt} = 0.2P(100 - P)$$

The formula for the solution $P = P(t)$ will depend on an initial value P_0 which is not specified.

Problem 2 Solve the differential equation

$$\frac{dP}{dt} = 0.04P(150 - P)(P - 30)$$

The formula for the solution $P = P(t)$ will depend on an initial value P_0 which is not specified.

Method for solving these equations: I will outline the method for solving Problem 1. Problem 2 is similar.

Separate the variables:

$$\frac{dP}{P(100 - P)} = 0.2dt$$

Then integrate. You need to figure out

$$\int \frac{dP}{P(100 - P)}$$

Note that it is NOT the case that every time when you integrate a fraction the result is the logarithm of the denominator.

In order to find this integral we use the method of *decomposing the fraction into simple fractions*. The simple fractions involved in this decomposition are

$$\frac{1}{P}, \frac{1}{100 - P}$$

(each factor of the original denominator appears as denominator in one of the simple fractions). We set up the original fraction as being equal

to a sum of these simple fractions, with unknown coefficients:

$$\frac{1}{P(100 - P)} = \frac{c}{P} + \frac{d}{100 - P}$$

We need to solve for the unknown coefficients c and d that would make this equality true. Bring the two fractions on the right hand side to their common denominator. The equation becomes

$$\frac{1}{P(100 - P)} = \frac{c(100 - P) + dP}{P(100 - P)}$$

Since the two sides now have the same denominator, in order for the fractions to be equal they must have the same numerator, so

$$1 = c(100 - P) + dP = (d - c)P + 100c$$

The equality will hold if $d - c = 0$ and $100c = 1$ (view the right hand side as a polynomial in the variable P ; set the coefficients of this polynomial equal to the coefficients of the polynomial on the left hand side - the coefficient of P is zero, and the constant term (with no P) is 1).

You should be able to complete the calculation from here on, and proceed with problem 2 along the same lines. Note that we know how to integrate each of the simple fractions $1/P$ and $1/100 - P$.