Summary of Competition and Predation Models

1. Competition

Assumption: There is a limited amount of resources that two populations compete for.

- each population grows logistically in the absence of the other one
- the presence of the each population affects the growth of the other one negatively

Equations:

$$\frac{dN_1}{dt} = r_1 N_1 \frac{K_1 - N_1 - \alpha N_2}{K_1}$$

$$\frac{dN_2}{dt} = r_2 N_2 \frac{K_2 - N_2 - \beta N_1}{K_2}$$

The state space: The two isoclines are the lines

$$N_1 + \alpha N_2 = K_1,$$
$$\beta N_1 + N_2 = K_2$$

The isoclines divide the state space into four regions. The short term behavior and the long run outcome depend on the region is where the initial value is.

Equilibia: There are three equilibria with at most one of the populations present: $(0, 0)$, $(K_1, 0)$, $(0, K_2)$.

Additionally, there might be one more equilibrium possible with both population s present, if the isoclines intersect. If the isoclines do not intersect, then it is not possible to have equilibrium with both populations present.

When the equilibrium with both populations present exists, it might be stable or unstable. If it is unstable, then the long term outcome of the competition will be one of the populations driving the other one to extinction. Then the remaining population will approach its carrying capacity. The winning population depends on the choice of the initial values. If the equilibrium with both populations present is stable, then the two populations will continue to coexist.

If there is no equilibrium with both populations present, then the long term outcome will be one of the populations driving the other to extinction. In this case, the winning population is the same regardless of where the inital values are. The winning population in this case is determined by the positions of the isoclines relative to each other.

2. Predation

Assumptions:

- The victims grow exponentially or logistically in the absence of the predators
- The predators decline exponentially in the absence of the victims
- The presence of the victims affects the growth of the predator population positively
- The presence of the predators affects the growth of the victim population negatively. The negative amount is equal to $R(V)P$, where $R(V)$ is the feeding rate of the predators.
- The feeding rate (or functional response) of the predators can be of type I (linear, $R(V) = \alpha V$), type II ($R(V) + \frac{kV}{V+D}$), or type III ($R(V) = \frac{kV^2}{V^2+D^2}$)

  The main features of a functional response of type II and type III are an almost-linear behavior when V is small, and a saturation amount (or maximum feeding rate) equal to k that is approached when V is large.

Equations:

$$\frac{dP}{dt} = -qP + \beta VP$$

— this is assumed in all the versions of the model

if victims grow exponentially in the absence of the predators, then:

$$\frac{dV}{dt} = rV - R(V)P$$

if victims grow logistically in the absence of the predators, then:

$$\frac{dV}{dt} = rV(1 - \frac{V}{K}) - R(V)P,$$

or

$$\frac{dV}{dt} = rV - cV^2 - R(V)P$$

The state space and equilibria: The two isoclines always intersect, which means tha there is always an equilibrium value with both populations present. If the equation of V is exponential when $P = 0$, then the equilibrium with both populations present (found where the isoclines intersect) and $(0, 0)$ are the only equilibria. If the equation of V is logistic when $P = 0$, then we have an additional equilibrium when $P = 0$ and $V =$ its carrying capacity.

The isocline for P is always a vertical line $V = q/\beta$. The isocline of V could be a horizontal line ($P = r/\alpha$), or a line with negative slope (if the equation for V is logistic when $P = 0$ and the feeding rate is αV), or a line with positive slope (if the equation for V is exponential when $P = 0$ and the feeding rate is type II).

The short term behavior is always cyclical. However the cycles might spiral inward or outward. If the cycles spiral outward they might hit one of the axes of coordinates so that one of the two species becomes extinct.