Math 172 Fall 2012 Handout 13

Predator-Prey Models

We study a predator-prey model under the following assumptions:

- In the absence of the predators, the victim population would follow a logistic model:
$$\frac{dV}{dt} = rV(1 - \frac{V}{K})$$
- In the absence of the prey, the predator population would decline exponentially:
$$\frac{dP}{dt} = -qP$$
- The amount of prey consumed by the predators is proportional to the sizes of the two populations.

The equations that reflect these assumptions are:
$$\frac{dV}{dt} = rV(1 - \frac{V}{K}) - \alpha VP$$
$$\frac{dP}{dt} = -qP + \beta VP$$
To find the equilibrium pairs we set
$$rV(1 - \frac{V}{K}) - \alpha VP = 0$$
$$-qP + \beta VP = 0$$
Each of the equations can be factored out as follows:
$$V(r - \frac{rV}{K} - \alpha P) = 0$$
$$P(-q + \beta V) = 0$$
There are two possibilities for the first equation to hold: $V = 0$ or $r - \frac{rV}{K} - \alpha P = 0$, which can be rewritten as $P = (r - \frac{rV}{K})/\alpha$. There are also two possibilities for the second equation to hold: $P = 0$ or $V = q/\beta$.

Equilibria with only one population present: if $V = 0$ then the only possibility for the second equation to be true is $P = 0$ (there can be no predators in the absence of the prey).

if $P = 0$, then the second eqution is automatically true, and for the first equation we need either $V = 0$ or $r - \frac{rV}{K} = 0$, which means $V = K$.

Thus the following equilibrium pairs are found when only one of the populations is present: $(0, 0)$, $(K, 0)$ (which were to be expected based on our assumptions).

When both populations are present we must have $V = q/\beta$ for the second equation to hold, and $P = (r - \frac{V}{K})/\alpha = (r - \frac{rq}{\beta K})/\alpha$ for the first equation to hold.

Note that the value of P found above could be positive or negative. If the value obtained from this expression, then we say that no equilibrium is possible with both populations present. Note that this situation occurs when $q/\beta > K$, in other words the value of V that would be needed to sustain the predator population at a constant level is larger than the carrying capacity of the victim population and thus this value cannot be maintained. The long term outcome in this situation is: the P population declines to extinction, the V population reaches carrying capacity K.