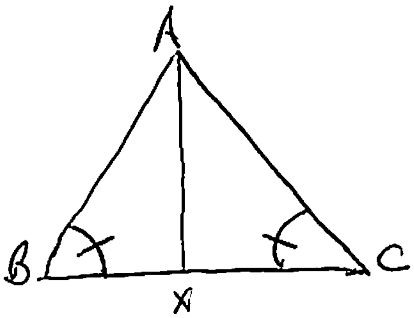
1B. Congruent Triangles

(1, 3, 5, 7, 8)

1. Suppose, in ΔABC, that $\angle B = \angle C$. Show that $AB = AC$

Proof:



Let $AX = \text{bis}(\angle A)$. Then we have $\angle BAX = \angle CAX$.

But also, we have $\angle B = \angle C$ (by hypothesis)

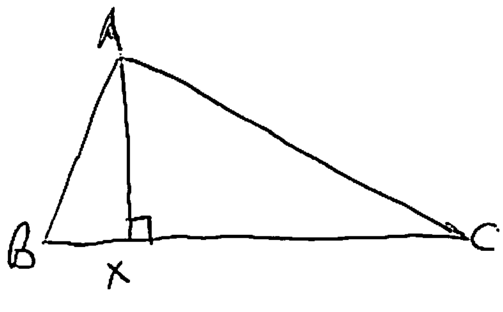
and $AX = AX$.

It follows by AAS that $\Delta ABX \cong \Delta ACX$. Hence, we see that

$AB = AC$ (corresponding sides).

3. Suppose, in ΔABC, that $\text{alt}(A) = \text{med}(A) = AX$. Show that $AB = AC$.

Proof:



We have:

$AX = \text{alt}(A) \Rightarrow \angle BXA = \angle CXA$
$AX = \text{med}(A) \Rightarrow BX = CX$ } hypotheses

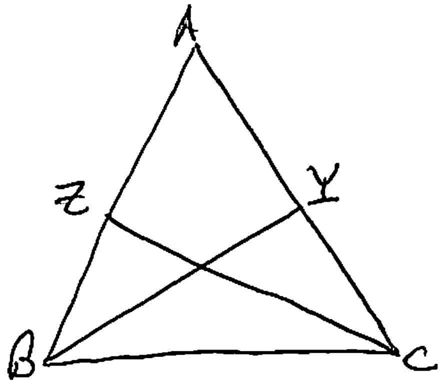
But also, we have $AX = AX$

$\xRightarrow{\text{SAS}} \Delta ABX \cong \Delta ACX$

$\Rightarrow AB = AC$

(corresponding sides)

5.



Suppose that ΔABC is isosceles with base BC, and that
- $BY = \text{bis} \angle B$
- $CZ = \text{bis} \angle C$.

Show that $BY = CZ$.

<u>Proof</u>: We have

$BY = \text{bis}(\angle B) \Rightarrow \angle B = 2\angle ABY$

$CZ = \text{bis}(\angle C) \Rightarrow \angle C = 2\angle ACZ$.

Now, since ΔABC is isosceles with base BC, we find that

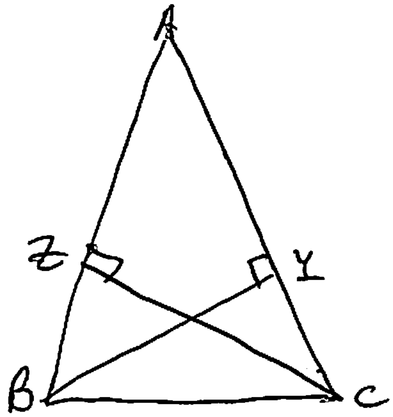
① $\angle B = 2\angle ABY = 2\angle ACZ = \angle C \xrightarrow{\div 2} \angle ABY = \angle ACZ$.

② $AB = AC$.

We also observe that $\angle A = \angle A$. By ASA, we conclude that

$\Delta ABY \cong \Delta ACZ$; therefore, we have $BY = CZ$.

7.



Suppose that

$BY = \text{alt}(B) = \text{alt}(C) = CZ$.

Show that $AB = AC$.

<u>Proof</u>: Consider the <u>following</u> triangles:

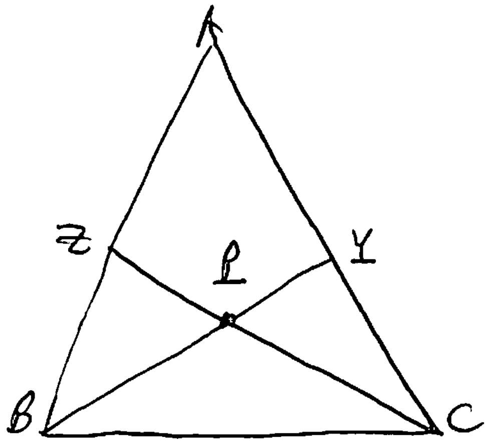
We have

$$\left.\begin{aligned} \angle Z &= \angle Y = 90^\circ \text{ (since } CZ, BY \text{ are altitudes)} \\ \angle A &= \angle A \\ CZ &= BY \text{ (by hypothesis)} \end{aligned}\right\} \overset{AAS}{\Longrightarrow} \triangle AZC \cong \triangle AYB$$

$$\Longrightarrow AC = AB$$

(corr. sides).

8.



Suppose that $BY = CZ$ and $PY = PZ$. Show that $AB = AC$.

Proof: We have

$$BP + PY = BY \overset{\text{by hypothesis}}{=} CZ = CP + PZ$$

$$PY = PZ \text{ (by hypothesis)}$$

subtract $\Longrightarrow$ $BP = CP$.

Consider triangles ΔZBP and ΔYCP.

$$\left.\begin{array}{l} \text{Vertical angles thm.} \Longrightarrow \angle BPZ = \angle CPY \\ \text{earlier work} \Longrightarrow BP = CP \\ \text{hypothesis} \Longrightarrow PZ = PY \end{array}\right\} \xrightarrow{\text{SAS}} \Delta ZBP \cong \Delta YCP.$$

$$\Longrightarrow \underbrace{\angle ZBP}_{= \angle ABY} = \underbrace{\angle YCP}_{= \angle ACZ} \text{ (corresponding angles)}$$

$$\left.\begin{array}{l} \\ \text{We also have } \angle A = \angle A \\ BY = CZ \text{ (hyp.)} \end{array}\right\} \xrightarrow{\text{AAS}} \Delta ABY \cong \Delta ACZ$$

$\Longrightarrow AB = AC$ (corresponding sides).