Final Examination – Math 531

Due Saturday, December 15

(1) Prove that a quadrilateral is a parallelogram if and only if its diagonals bisect each other.

(2) Let AX be the angle bisector of $\angle A$ in $\triangle ABC$. Prove that

$$\frac{BX}{XC} = \frac{AB}{AC}. \tag{1}$$

(3) Let P be a point inside a circle, and let XY and UV be two chords going through P. Prove that $PX \cdot PY = PU \cdot PV$.

(4) Given acute angled $\triangle ABC$ as in Figure III.1, extend the altitudes from A, B, C to meet the circumcircle at X, Y, and Z respectively. Prove that AX bisects $\angle ZXY$.

Hint. Look for congruent triangles.

(5) Suppose that AX is a median of a triangle, and G is the centroid. Prove that $AG = 2GX$.

(6) Prove that the three angle bisectors of a triangle are concurrent.

(7) Let A, B, C, D be four collinear points and suppose that P is a point not on the line through them. Prove that

$$cr(A, B, C, D) = \frac{\sin(\angle APC)\sin(\angle BPD)}{\sin(\angle APD)\sin(\angle BPC)}.$$