## Homework 8 - Math 141, Frank Thorne (thornef@mailbox.sc.edu) Due Friday, October 14

These questions will not appear on the second midterm exam.
(a) What does Rolle's Theorem say? Draw a picture and explain.
(b) What does the Mean Value Theorem say? Draw a picture and explain.
(c) You drive 120 miles from Columbia to Charlotte, and it takes you exactly two hours. At some point, your speed is exactly 60 mph . Explain how you know this. Must there be some point at which your speed is exactly 65 mph ?
(d) (Careful! You have to think about this one...) In the question above, must there be some point at which your speed is exactly 55 mph ?
(e) You throw a ball straight up in the air, and eventually it falls back to the earth. Is there some time in its flight when the ball is not moving at all? Explain.
(f) Consider the function $f(x)=|x|$. Find its derivative. We have $f(1)=f(-1)=0$, but show that there is no point $x$ for which $f^{\prime}(x)=0$. Explain why this doesn't contradict the Mean Value Theorem. Draw a picture which illustrates your conclusions.
(g) Consider the function $f(x)=x^{2}$. Use the Mean Value Theorem to show that there is a $c \in(-1,2)$ with $f^{\prime}(c)=1$. Then, find $f^{\prime}(x)$ and figure out all possible values of $c$. Draw a picture which illustrates your conclusions.
(h) Consider the function $f(x)=\sin (x)$. Use the Mean Value Theorem to show that there is a $c \in(0,4 \pi)$ with $f^{\prime}(c)=0$. Then, find $f^{\prime}(x)$ and figure out all possible values of $c$. Draw a picture which illustrates your conclusions.
(i) Consider the function $f(x)=\sin (x)$. Use the Mean Value Theorem to show that there is a $c \in(0,5 \pi / 2)$ with $f^{\prime}(c)=2 / 5 \pi$. Then, find $f^{\prime}(x)$ and figure out, approximately, all possible values of $c$. Draw a picture which illustrates your conclusions.
(j) Stewart, Ch. 4.2, 7-8.
(k) Stewart, Ch. 4.2, 1-4, 11-14; even required, odd recommended.

