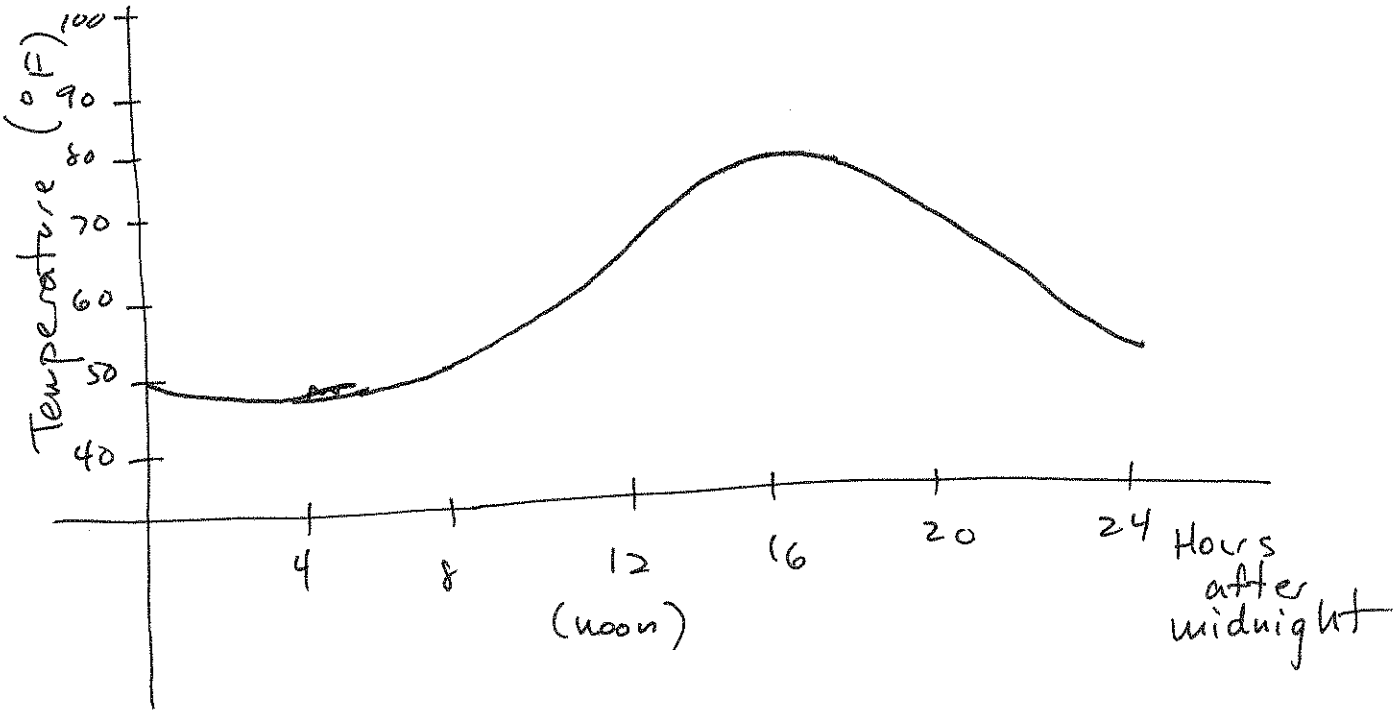
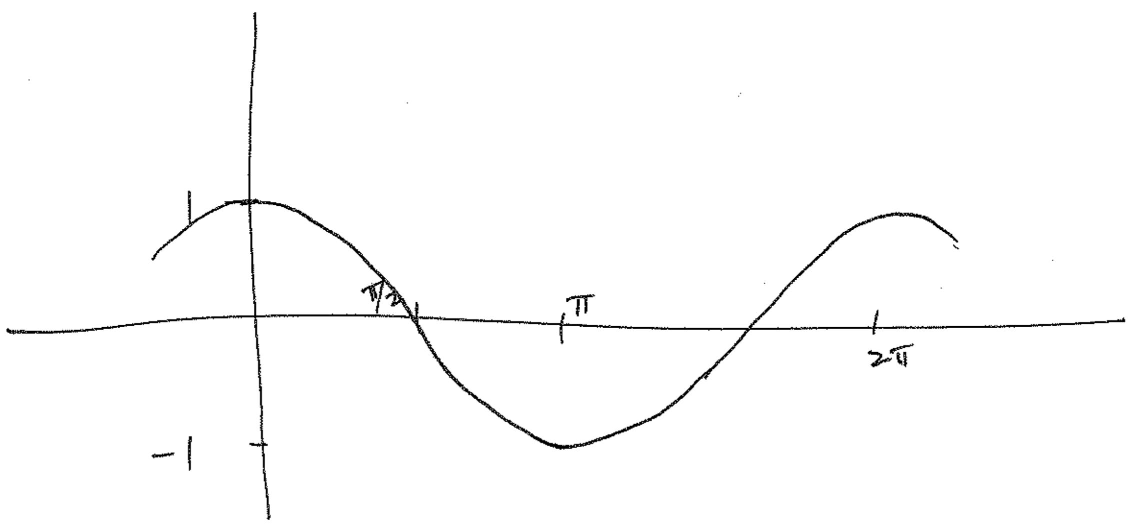
1. (10 points)



2. (12 points)

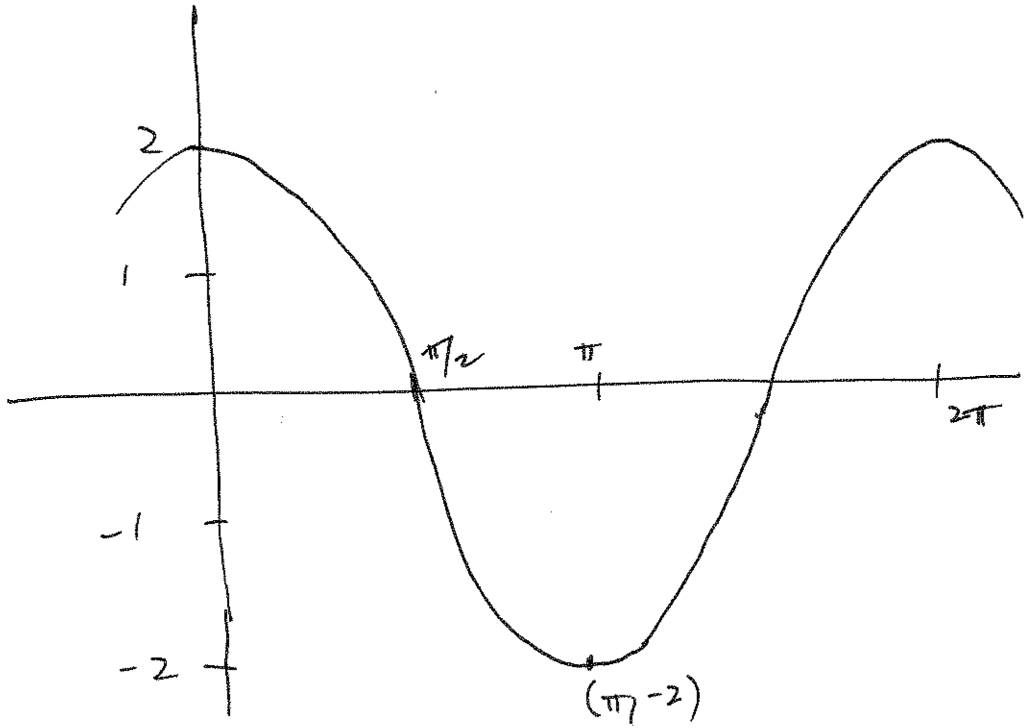
The graph of $y = \cos x$ is



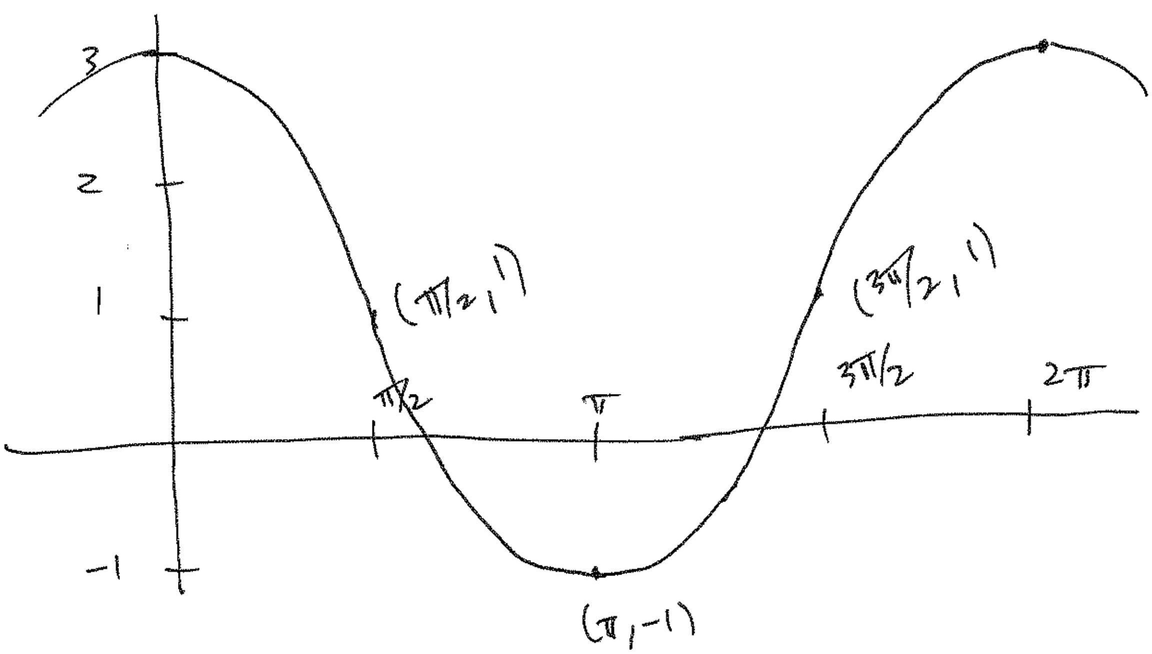
If (u, v) is a point on $y = \cos x$

then $(u, 2v)$ is a point on $y = 2\cos x$.

So we vertically stretch the graph by a factor of 2:



If (u, v) is a point on $y = 2\cos x$
then $(u, v+1)$ is a point on $y = 2\cos x + 1$.
So we stretch the graph up by 1!



3. (12 points)

(Good answer) $\lim_{x \to 2} f(x) = 5$ means that as x gets closer and closer to 2, f(x) gets closer and closer to 5.

(better answer) As x gets close and closer to 2, f(x) can be made arbitrarily close to 5.

It is possible for this to be true and yet $f(2) = 3$, because $\lim_{x \to 2} f(x)$ depends only on the values of f(x) near $x = 2$, and not actually at 2 itself.

4. (12 points) $\lim_{x \to -1} \frac{x^2 - 4x}{x^2 - 3x - 4} = \lim_{x \to -1} \frac{x(x-4)}{(x-4)(x+1)}$

$= \lim_{x \to -1} \frac{x}{x+1}$

If we plug in $x = -1$ we get $\frac{-1}{0}$.

As x gets closer to -1 this expression blows up and so the limit does not exist.

5. (12 pts.) $\lim_{x \to -\infty} \frac{1 - x - x^2}{2x^2 - 7} = \lim_{x \to -\infty} \frac{\frac{1}{x^2} - \frac{x}{x^2} - \frac{x^2}{x^2}}{\frac{2x^2}{x^2} - \frac{7}{x^2}}$

$= \lim_{x \to -\infty} \frac{\frac{1}{x^2} - \frac{1}{x} - 1}{2 - \frac{7}{x^2}}$.

As $x \to -\infty$, $\frac{1}{x} \to 0$ and so this limit is $\frac{0 - 0 - 1}{2 - 0} = \frac{-1}{2}$.

6. $\displaystyle \lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2+x} \right)$

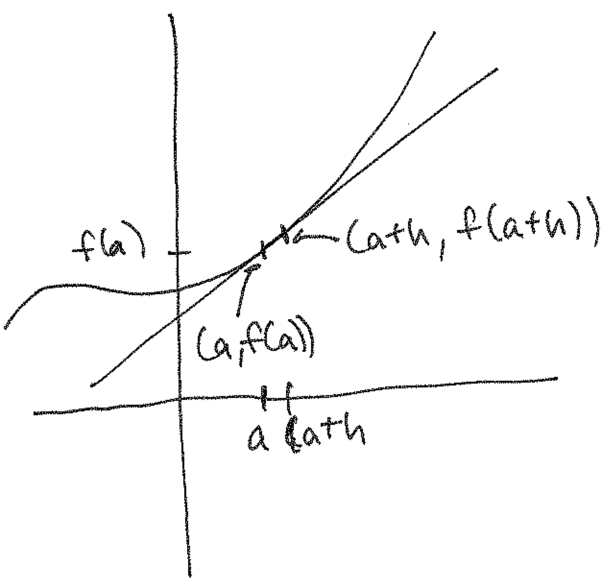
$$= \lim_{x \to 0} \frac{x^2 + x - x}{x(x^2+x)}$$

$$= \lim_{x \to 0} \frac{x^2}{x(x^2+x)} = \lim_{x \to 0} \frac{x^2}{x^2(x+1)}$$

$$= \lim_{x \to 0} \frac{1}{x+1}$$

$$= \frac{1}{0+1} = 1.$$

7. The derivative of $f(x)$ at $x=a$ is $\displaystyle \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.



If we pick a small value of h, then the slope of the secant line between the points $(a, f(a))$ and $(a+h, f(a+h))$ is $\frac{\text{rise}}{\text{run}}$ or $\frac{f(a+h) - f(a)}{h}$. As h gets closer to 0, this secant line gets closer to the tangent line and its slope gets closer to the derivative. Therefore, when we take the limit as $h \to 0$, the derivative gives the slope of the tangent line.

8. We could compute $f'(x)$ in general.
But this is shorter:

By definition,

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{(1-h^3) - (1-0^3)}{h}$$

$$= \lim_{h \to 0} \frac{-h^3}{h} = \lim_{h \to 0} -h^2 = -0^2 = 0.$$

The slope of the tangent line is 0.
Since this line goes through $(0,1)$, its equation is

$$(y-1) = 0 \cdot (x-0)$$

$$y - 1 = 0$$

$$\boxed{y = 1}.$$