NAME:

MATH 141 FIRST TEST

THIS IS A CLOSED BOOK, CLOSED NOTES TEST, USE OF CALCULATORS IS NOT ALLOWED. IF MORE SPACE NEEDED, LEAVE A MARK AND CONTINUE ON THE OTHER SIDE OF THE PAPER. SHOW DETAILS OF YOUR WORK.

1) Differentiate the following functions:

(a) $f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ 4

(b) $f(x) = e^{\cos x}$ 4

c) $f(x) = \sin(x^2 + 2)$ 4

d) $f(x) = \frac{\ln(x^2+2)}{x}$ 4

2) (a) State the Mean Value Theorem. 5

(b) Give the chain rule in Leibniz form, when z is a function of y, y is a function of x, and x is a function of t. $\frac{dz}{dt} =$ 4

3) Do implicit differtiation.

a) $x^2 + y^2 = 25$ is the equation of a circle. What is the slope of the tangent in $(3, 4)$? 3

b) $x^3 + y^3 = 9$ is the equation of a curve. What is the slope of the tangent in $(2, 1)$? . 3

4) Find absolute maximum and minimum values of the function $f(x) = \sin x$ in the interval $[0, \pi]$ using the Closed Interval Algorithm. 8

5) Find a c value corresponding to the mean value theorem for $f(x) = \frac{x^3}{3}$, $[a, b] = [0, 4]$.
. 10

6) Given $f(x) = x^3 - 4x$.
a) Find the increasing and decreasing intervals of $f(x)$. 8

b) Find the local extreme values of $f(x)$. Where is local minimum, local maximu, and what are the values? 8

7) Given $f(x) = \frac{x^4}{12} - \frac{x^2}{2}$. Find concave up and concave down intervals of $f(x)$. Is there any point of inflection?

7

8) Consider the function $f(x) = x \sin^2 x$.
a) Verify that $f'(0) = 0$. 4

b) Decide whether $f(x)$ has a local extreme value at $x = 0$. 4

9) We know that $f' > 0$ in $(-\infty, -1)$, $f' < 0$ in $(-1, 2)$, and $f' > 0$ in $(2, \infty)$. We also know that $f" < 0$ in $(-\infty, 0)$, and $f" > 0$ in $(0, \infty)$. We also know that $f(-1) = 1, f(0) = 0, f(2) = -2$. Sketch the graph of the function. 14

10)a) Find all asymptotes of the function $f(x) = x + \frac{1}{x}$. 5

Find all asymptotes of the function $f(x) = \frac{1}{\sin x}$. 5

BONUS PROBLEM Does $f(x) = x \sin^2 x$ from Problem 8 have an inflection point at $x = 0$?