Name _____

June 20, 2008

Math 574
Quiz 11
Show Your Work

1. Determine the number of strings of length n using the four letters a, b, c, and d that contain each letter at least once.

   Solution: Use Inclusion – Exclusion. Let S denote the set of all strings of length n using the letters a, b, c, and d.

   Let
   A_1 denote the set of all strings of length n that do not contain the letter a.
   A_2 denote the set of all strings of length n that do not contain the letter b.
   A_3 denote the set of all strings of length n that do not contain the letter c.
   A_4 denote the set of all strings of length n that do not contain the letter d.

   Then we want the value of
   $$\left| \overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4} \right| = N_0 - N_1 + N_2 - N_3 + N_4 = 4^n - 4 \cdot 3^n + 6 \cdot 2^n - 4 .$$

2. How many derangements of the numbers 1, 2, 3, 4, 5, 6, 7 have each even integer in an even position? *Simplify* your answer to an integer.

   Solution: Since the even integers must be in the even positions, the odd integers must be in the odd positions.

   So, first place the three even integers in the three even positions so that no integer is in its proper position, and then do the same thing with the four odd integers. You get, $D_3 \times D_4 = 9 \times 44 = 396$.