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Math 574
Exam 3
Show All Work!

1. Find a recurrence relation along with appropriate initial conditions for the number c_n of n-digit numbers using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 such that the sum of the digits is even. Simplify the recurrence relation so that no subscript appears more than once.

2. (a). Using Inclusion-Exclusion, determine the number of permutations of 1, 2, 3, 4, 5 6, 7, 8, 9 in which each of 1, 2, 3, and 4 is not in its proper position. *Clearly define the sets that you are using*.

   (b). How many functions from $\{a, b, c, d, e\}$ to $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ use each even integer as a value at least once?

Name ______________________________

3. Let c_n denote the number of strings of length n using the letters a, b, c, d, e that have at least one occurrence of the pattern *ab or* at least one occurrence of *aa*. Find a recurrence relation for c_n along with appropriate initial conditions. Justify your work! Explain how you got your relation.

4. (a). How many permutations of the digits 1, 2, 3, 4, 5, 6, 7 have exactly 3 numbers in their proper position? [Simplify your answer to an integer.]

   (b). How many derangements are there of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 10 such that every even integer is in an even position? [Simplify your answer to an integer.]

   (c). Simplify $p_n - p_{n-1}$ to a simplified fraction (in lowest terms) in terms of n. [p_n is the probability that a random permutation is a derangement.] Justify your answer.

Name ______________________________

5. Suppose we are making a walkway using Red, Blue, Yellow and Green blocks. The Red and Blue blocks are 1-inch long and the Yellow and Green blocks are 2-inches long. We are not allowed to use two consecutive Red blocks. Let w_n denote the number of possible walkways of length n that we can form.

   (a). Find a recurrence relation for w_n along with appropriate initial conditions.

   (b). How many possible walkways are there of length 6?

6. Find a formula for the solution to $a_0 = 1$, $a_1 = 3$, and for $n \geq 2$, $a_n = 6a_{n-1} - 8a_{n-2}$.

Name ______________________________

7. How many strings of length 8 using the letters *a, b, c, d* do not contain exactly one occurrence of any letter? i.e., each letter either does not occur at all or it occurs at least twice.
Example: *a b b c c c b b* is bad because there is just one *a*.
and *b c c c c d a c* is bad because *b, a* and *d* all occur just once.
However, *a b b d a a d d* is OK – note that *c* does not occur at all in this string.

8. Show that if each of the line segments joining 66 points is colored red, blue, green, or yellow, then there must exist some triangle (using the given points as its vertices) all of whose sides have the same color.

9. How many subsets of $S = \{a, b, c, d, e, f, g, h\}$ intersect each of the sets $\{a, b\}$, $\{c,d\}$, $\{e,f\}$, $\{g,h\}$? i.e., How many subsets of S contain at least one of a or b, at least one of c or d, at least one of e or f, and at least one of g or h?
Example: $\{a, c, d, f, h\}$ is OK but $\{a, b, e, f, g, h\}$ is not since it does not intersect $\{c, d\}$.