Math 250
Problem Set 19

1. Set up the integral below using polar coordinates [You need not evaluate].

   $\int_1^2 \int_0^{\sqrt{2x-x^2}} x^2 + y^2 dy\, dx$.

   Solution: Done in class.

2. (a). Determine the spherical coordinates of the point having rectangular coordinates $\left(2\sqrt{3},\, 6,\, 4\right)$. Note that this is a point in the first quadrant.

   Solution: Done in class.

   (b). Convert the equation $\rho = \sec\phi$ into rectangular coordinates and describe what kind of surface it is. (It is *not* a sphere!)

   Solution: Multiply by $\cos\phi$ on both sides and simplify to get $z = 1$.

3. (a). If a point has spherical coordinates $\rho = 4$, $\phi = \pi/6$, and $\theta = \pi/3$ then

   (i). Determine the rectangular coordinates (x, y, z))

   Solution: $\left(1, \sqrt{3}, 2\sqrt{3}\right)$

   (ii). Determine the cylindrical coordinates (r, θ, z).

   Solution: $\left(2, \dfrac{\pi}{3}, 2\sqrt{3}\right)$

   (b). Change the equation $\rho = 2\sin\phi\cos\theta$ to rectangular coordinates and describe its graph

   Solution: Multiply both sides by ρ to get,

   $\rho^2 = 2\rho\sin\phi\cos\theta \Leftrightarrow x^2 + y^2 + z^2 = 2x \Leftrightarrow (x-1)^2 + y^2 + z^2 = 1$.

   So, this is a sphere of radius 1 and center at the origin.

4. Express the volume of the solid lying below the sphere $x^2 + y^2 + z^2 = 13$ and above the plane $z = 2$ as an iterated triple integral. *Don't integrate.*

   Solution: Done in class.

5. Show that the integral $\int_0^\infty e^{-x^2} dx$ converges.

   Solution: This follows from the fact that $F(b) = \int_0^b e^{-x^2} dx$ is an increasing function of $b \geq 0$ and that $F(b)$ is bounded from above because $\int_1^\infty e^{-x^2} dx \leq \int_1^\infty e^{-x} dx = 1$ [Note that for $x \geq 1$, $e^{-x^2} \leq e^{-x}$].

Section 5.5 #13, 14, 21, 27

Review Exercises (Page 365): #9, 13,