

Math 142
Exam #3
Show All Work

For each series in problems 1 – 5, Apply an appropriate convergence test to determine if the series converges or diverges. Explain your work clearly, naming the test being used, and fully evaluating any integrals and/or limits completely and explain their significance.

1.
$$\sum_{n=1}^{\infty} \frac{(2n-1)!}{5^n (n!)^2}$$

2. (a).
$$\sum_{n=1}^{\infty} \frac{n^{3n+1}}{(2n+1)^n}$$

(b).
$$\sum_{n=1}^{\infty} \frac{1}{(\tan^{-1} n)^2}$$

3. (a). Evaluate the sum:
$$\sum_{n=2}^{\infty} \frac{2^{n+1} + 5}{2^{3n-2}} = \underline{\hspace{2cm}}$$

(b). Evaluate the limit:
$$\lim_{n \rightarrow \infty} \frac{n^{2n}}{n+3}$$

4. $\sum_{n=1}^{\infty} \frac{2n+1}{\sqrt{n^6+3n^2+5}}$

5. Use the integral test to determine if $\sum_{n=1}^{\infty} \frac{n}{e^n}$ converges.

6. (a). Evaluate the n th partial sum $\sum_{k=1}^n \frac{k+1}{k+2} - \frac{k}{k+1} =$ _____ (An expression in n)

(b). What is the value of $\sum_{k=1}^{\infty} \frac{k+1}{k+2} - \frac{k}{k+1} =$ _____ (a number or infinity)

7. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n \sqrt{n}}$.

8. (a). Find the series expansion of $\frac{\sin x + \cos x}{e^{-x}}$ to terms of degree 3.

(b). Given that $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{3}{16}x^3 - \frac{5}{128}x^4$, find the series expansion of $\frac{1}{\sqrt{1+2x}}$ to terms of degree 3. (**Hint:** $\frac{1}{\sqrt{1+x}} = 2 \frac{d}{dx} [\sqrt{1+x}]$?)

9. Find the Taylor polynomial of degree 4 around $x = 1$ of $f(x) = \sqrt{x}$.