Name ______________________

June 30, 2003

Math 142
Exam #3
Show All Work

For each series in problems 1 – 5, Apply an appropriate convergence test to determine if the series converges or diverges. Explain your work clearly, naming the test being used, and fully evaluating any integrals and/or limits completely and explain their significance.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n-1)!}{5^n (n!)^2}$

2. (a). $\displaystyle\sum_{n=1}^{\infty} \left[\ln\left(\frac{3n+1}{2n+1}\right)\right]^n$

   (b). $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\left(\tan^{-1} n\right)^2}$

3. (a). Evaluate the sum: $\displaystyle\sum_{n=2}^{\infty} \frac{2^{n+1} + 5}{2^{3n-2}}$ = ____________________

   (b). Evaluate the limit: $\displaystyle\lim_{n\to\infty} \left(\frac{n}{n+3}\right)^{2n}$

Name ________________________

June 30, 2003

4. $\sum_{n=1}^{\infty} \frac{2n+1}{\sqrt{n^6 + 3n^2 + 5}}$

5. Use the integral test to determine if $\sum_{n=1}^{\infty} \frac{n}{e^n}$ converges.

Name ________________________

June 30, 2003

6. (a). Evaluate the nth partial sum $\displaystyle\sum_{k=1}^{n}\left(\frac{k+1}{k+2}-\frac{k}{k+1}\right)=$ ________ (An expression in n)

(b). What is the value of $\displaystyle\sum_{k=1}^{\infty}\left(\frac{k+1}{k+2}-\frac{k}{k+1}\right)=$ ____________ (a number or infinity)

7. Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty}\frac{(x-2)^n}{3^n\sqrt{n}}$.

Name ______________________

June 30, 2003

8. (a). Find the series expansion of $\dfrac{\sin x + \cos x}{e^x}$ to terms of degree 3.

(b). Given that $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{3}{16}x^3 - \frac{5}{128}x^4$, find the series expansion of $\dfrac{1}{\sqrt{1+2x}}$ to terms of degree 3. (**Hint**: $\dfrac{1}{\sqrt{1+x}} = 2\dfrac{d}{dx}\left[\sqrt{1+x}\right]$?)

9. Find the Taylor polynomial of degree 4 around $x = 1$ of $f(x) = \sqrt{x}$.