The Diameter of Domination k-Critical Graphs

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Abstract

A graph is k-domination-critical if $\gamma(G)=k$, and for any edge e not in G, $\gamma(G+e) = k-1$. In this paper we show that the diameter of a domination k-critical graph with $k\geq2$ is at most 2k-2. We also show that for every $k\geq2$, there is a k-domination-critical graph having diameter $\left[\frac{3}{2}k-1\right]$. We also show that the diameter of a 4-domination-critical graph is at most 5.

I Introduction

Several authors have studied the concepts of graphs that are critical with respect to various operations on graphs. See in particular Bauer, Harary, Nieminen and Suffel [1] which deals with the behavior of the domination number with respect to edge deletion, and Brigham, Chinn and Dutton [2] which deals with the effect upon the domination number of deleting vertices.

Sumner-Blitch [4] introduced the concept of a k-domination-critical graph in the sense of edge addition. Wojcicka [6] showed that any connected 3-domination-critical graph on more than 7 vertices is traceable. In Sumner-Blitch [4], the diameter of a connected, 3-domination-critical graph was shown to be at most three.

In this paper we investigate the problem of the maximum possible diameter of a k-domination-critical graph for $k \geq 4$. We will show that the diameter of a connected, 4-domination-critical graph is at most 5, and that for $k\geq5$, the diameter of a connected, k-domination-critical graph is at most 2k-2. We show that for each $k\geq2$, there is a connected, k-domination-critical graph having diameter $\left[\frac{3}{2}k-1\right]$.

II Preliminaries

All our graphs will be simple having no loops nor multiple edges. The distance between the vertices v and u will be denoted by d(v,u). We will denote the diameter of the graph G by diam(G). We will let N(v)={u: u is adjacent to v} denote the neighborhood of v and $N[v] = N(v) \cup \{v\}$ denote the closed neighborhood of v.

Suppose that A and B are sets of vertices in the graph G. Then we say that A *dominates* B iff every vertex of B-A is adjacent to some vertex of A. If A dominates V(G), then we say that A is a *dominating set* for G. The cardinality of a smallest dominating set in G is called the *domination number* of G, and is denoted by $\gamma(G)$.

When no confusion is possible, we will not distinguish between a set A of vertices of a graph G, and the subgraph induced by A. In particular, if A is a subset of V(G), then we will write just $\gamma(A)$ for the domination number of the subgraph induced by A.

A graph G is *k-domination-critical* if $\gamma(G) = k$, and for every non-adjacent pair of vertices v and u, $\gamma(G+vu) = k-1$; thus a domination-critical graph is one for which the domination number drops whenever a new edge is added. For simplicity, we will henceforth write just *k-critical* instead of k-domination-critical.

The Arrow Notation

The following notation will be used throughout this paper.

If v and u are non-adjacent vertices in a k-critical graph G, then there must exist a set S of cardinality k-2 such that either $S \cup \{v\}$ dominates all of G-u, or $S \cup \{u\}$ dominates all of G-v. In the first case we write $[v, S] \rightarrow u$, and in the second we write $[u, S] \rightarrow v$. In particular, when we write $[v, S] \rightarrow u$ it is understood that u is <u>not</u> dominated by $S \cup \{v\}$. We will write $v \rightarrow u$ to indicate that a such a set S exists with $[v, S] \rightarrow u$.

Note: $v \to u$ means that there exists a (k-1)- element set S that contains v and dominates all of G-u.

The next two theorems are from Sumner-Blitch [4] . Generally, the removal of a vertex from a graph may cause the domination number to grow dramatically. This can not happen in k-critical graphs as Theorem 1 shows.

Theorem 1. If G is a k-critical graph, then for any vertex v of G, $\gamma(G\text{-}v) \leq k$.

Theorem 2. The diameter of a connected, 3-critical graph is at most 3.

The set $G^* = \{v \in V(G) : \gamma(G\text{-}v) = k\}$ of vertices whose removal does not alter the domination number of G has numerous useful properties. We refer to G^* as the *nucleus* of G. The darkened vertices in the graphs of Figure 1 represent the nucleus of each graph.

The next theorems are proven in Sumner-Wojcicka [5].

Theorem 3. If v is a cut-vertex of the connected, k-critical graph G, then $v \in G^*$.

Theorem 4. For any connected, k-critical graph G, if G^* is non-empty, then G^* is complete.

III. The Diameter of a Connected, k-Domination-Critical Graph

Lemma 1. If $x \to y$ for some vertices x and y of the connected, k-critical graph G, then y can not be a cut-vertex of G.

Proof: If $x \to y$, then $\gamma(G\text{-}y) = k\text{-}1$. So $y \notin G^*$, and hence by Theorem 3, y can not be a cut-vertex of G.♦

Let G be a connected, k-critical graph having diameter n, and let a be the terminal vertex of some longest path of G. Then the family $W = \{V_0, V_1, \ldots, V_n\}$, where $V_i = \{x: d(a,x) = i\}$ is called the *level-decomposition* of G with respect to a. In particular, $V_0 = \{a\}$. If A and B are two levels of this level-decomposition, then we say A is less than B (and write A < B) if $A=V_i$ and $B=V_j$ for i < j.

If v belongs to some V_i with 0<i<n and v dominates all of $V_{i-1} \cup V_i \cup V_{i+1}$, then we say that v is a *level-dominant* vertex. Also if V_i for 0<i<n contains a level-dominant vertex, then we refer to V_i as a *dominant level*.

Lemma 2. If v is a level dominant vertex of the connected, k-critical graph G, then v belongs to G*.

Proof: Suppose that $v \in V_i$ is level dominant, and that v does not belong to G*. Then γ(G-v)=k-1, and so there exists a (k-1)-element dominating set S of G-v. Now S can not dominate v, so it must be that no element of S belongs to $V_{i-1} \cup V_i \cup V_{i+1}$. Thus no vertex of S can dominate any vertex of V_i-{v} and so we have V_i = {v}. Hence, v is a cut vertex of G, and so by Theorem 3 we have a contradiction. ♦

Since G* is complete, the next result follows at once.

Corollary 1. There are at most two dominant levels in any level-decomposition of a connected, k-critical graph G, and furthermore any two dominant levels must be consecutive.

Lemma 3. Suppose that G is a k-critical graph, and $W = \{V_0, V_1, \ldots, V_n\}$ is a level-decomposition of G with respect to the vertex a. Let T be any set of t vertices none of which are level-dominant. Then T can dominate at most 2t levels of W.

Proof: Clearly this is true if t=1. We argue by induction.

Suppose that the result holds for values less than some t > 1, and let $T = \{x_1, x_2, \cdots, x_t\}$ be a set of t vertices. Suppose that T dominates the 2t+1 levels $A_1, A_2, \cdots, A_{2t+1}$ (so each A_i is some V_j). Assume that the A_i's are ordered so that if $A_i = V_j$ and $A_r = V_s$ with j < s, then i < r. Also assume that the x_i's are ordered so that for i<j, $d(a,x_i) \leq d(a,x_j)$.

We first observe that x_1 belongs to A_2. For suppose that x_1 does not belong to A_2. Then in order to dominate the vertices of A_1, it must be that x_1 can not dominate A_3. But then, the t-1 elements of T - $\{x_1\}$ would dominate the 2t-1 levels $A_3, A_4 \cdots, A_{2t+1}$ contrary to our inductive assumption. By a similar argument we observe that x_i belongs to A_{2i} for each i=1, 2,$\cdots$, t. In fact, suppose that x_j does not belong to A_{2j} and j > 1 is the first index for which this happens. Then x_j either belongs to a level less than A_{2j} or to to a level greater than A_{2j}. If x_j belongs to a level less than A_{2j} , then the t-j vertices x_{j+1}, x_{j+2},... x_t must dominate all of the 2(t-j)+1 levels A_{2j+1}, A_{2j+2}, ..., A_{2t+1}. On the other hand, if x_j belongs to a level greater than A_{2j}, then the j-1 vertices x_1, x_2,... x_{j-1} must dominate all of the 2(j-1)+1 levels A_1, A_2, ..., A_{2j-1}. It also follows from this observation that each x_i must dominate all of the vertices in A_{2i} (and x_1 must dominate A_1 as well.) Also it follows that the A_i's are consecutive levels, i.e., $d(a,A_{i+1}) = d(a,A_i)+1$.

Now x_1 is not adjacent to x_2, and so there exists a set S of k-2 vertices such that either $[x_1,S] \rightarrow x_2$ or $[x_2,S] \rightarrow x_1$.

Suppose first that $[x_1,S] \rightarrow x_2$. Then since x_1 must dominate all of A_1 and A_2, it can not also dominate all the vertices of A_3. Thus there is some vertex w in A_3 that is not adjacent to x_1. Now there must exist a vertex z in S that dominates w. Also, since z can not be adjacent to x_2, it must lie in $A_2 \cup A_3$. However, $\{x_1, x_2\}$ dominates all vertices in $A_1 \cup A_2 \cup A_3 \cup A_4$, and so the set $(S - \{z\}) \cup \{x_1, x_2\}$ is a k-1 element set that dominates all of G. This is impossible.

So it must be that there exists a set S with cardinality k-2 and $[x_2,S] \rightarrow x_1$. Now by Lemma 1, x_1 can not be a cut-vertex and so there must be some vertex w in A_2 - $\{x_1\}$.

There must be some vertex z in S that dominates w, and z must not be adjacent to x_1. This means that z lies in A_3. But, as before, we now see that $(S-\{z\})\cup\{x_1,x_2\}$ is a (k-1)-element dominating set for G. With this contradiction, the lemma follows. ♦

Theorem 5. For k≥2, the diameter of a connected, k-critical graph is at most 2k-2.

Proof. Since the result is known for k<4, we will assume that k ≥ 4. Let G be a k-critical graph having diameter n. Let a and b be vertices of G such that d(a,b) = n. Without loss of generality, we may assume there exists a set S of k-2 vertices such that $[a,S]\to b$. Let $W=\{V_0,V_1,\ldots,V_n\}$ be a level-decomposition for G with respect to a. Now

suppose that there are no dominant levels with respect to W. Then $S\cup\{a\}$ is a set of k-1 vertices that dominates all of the levels of W except possibly for V_n. Hence it follows from Lemma 3 that n ≤ 2(k-1).

Now suppose that there is exactly one dominant level V_i in W. Let $x \in V_{i-1}$ and $y \in V_{i+1}$. Then x is not adjacent to y, so there exists a set R of k-2 elements such that (without loss of generality) $[x,R]\to y$. But then, $R\cup\{x\}$ is a (k-1)-element set - no elements of which are level-dominant (since y is not dominated) - that dominates all levels of W except for V_{i+1}. Hence we have n ≤ 2k-2.

Finally, suppose that W contains two dominant levels, say V_i and V_{i+1}. Let x be a vertex in V_{i-1} and y a vertex in V_{i+2}. Then without loss of generality, there exists a set T of cardinality k-2 such that $[x,T]\to y$. Now if T has no level dominant vertices, we argue just as before. So suppose that T does contain a level-dominant vertex z (of course we may assume there is just one level dominant vertex in T as the argument is even simpler otherwise). Then since z can not be adjacent to y, z must belong to V_i. But now, the k-3 elements of T-{z} must account for all n+1 levels of W except possibly V_{i-2}, V_{i-1}, V_i, V_{i+1} and V_{i+2} (n-4 levels in all). Hence we have $2(k-3) \geq n-4$ or $n \leq 2k-2$. ♦

IV. A Class of k-critical Graphs Having Large Diameter.

We next give a simple construction that frequently makes it possible to extend a k-critical graph to a larger one. Let G be a graph, $v \in V(G)$, $v' \notin V(G)$. Then $\langle G,v,v' \rangle$ will denote the graph with $V(\langle G,v,v' \rangle)=V(G) \cup \{v'\}$, and $E(\langle G,v,v' \rangle)=E(G) \cup \{ v'u : u \in N[v]\}$. Thus $\langle G,v,v' \rangle$ is obtained from G by adding a new vertex v' that has the same closed neighborhood as v. We will call $\langle G,v,v' \rangle$ the *expansion of G via v (by v')*. If the expansion of the k-critical graph G via v is also k-critical then v will be called *expandable*. Figure 2 illustrates this construction.

The next lemma is proven in Sumner-Wojcicka [5]

Theorem 6. If v is a vertex of G such that $v \rightarrow u$ for every vertex $u \notin N[v]$, then v is expandable.

Definition: A graph G with $\gamma(G)=k$ is said to be *k-domination-symmetric* if for every two non-adjacent vertices x,y of G, both $x \rightarrow y$ and $y \rightarrow x$.

We obtain the next result as an easy consequence of Theorem 6.

Lemma 4. If G is k-domination-symmetric, then every vertex of G is expandable.

Now for purposes of this paper, we will refer to a graph G as being *k-good* if it satisfies:

1. G is k-domination-symmetric.
2. There exist two vertices a, b of G with

i. $\delta(a) = \delta(b) = 2$.

ii. N(a) and N(b) are both independent.

iii. d(a, b) = diam(G).

We will refer to any such vertices a and b as *diametric vertices* of G.

Note that the graph G_2 in Figure 3 is 2-good, and the graph G_3 in Figure 3 is 3-good.

Definition: Let G_1 be a k_1-good graph and G_2 a k_2-good graph with a_1 , b_1 diametric vertices of G_1 and a_2 , b_2 diametric vertices of G_2. Let $H_1 = \langle G_1,b_1,b_1'\rangle$ and $H_2 = \langle G_2,a_2,a_2'\rangle$. Form the graph G by taking disjoint copies of H_1 and H_2 and adding the edges b_1a_2 and $b_1'a_2'$. We refer to the graph G as the *symmetric sum* of G_1 and G_2. This construction is illustrated in Figure 4.

Lemma 5. Let $k \geq 2$, and let G be a k-good graph having a diametric vertex b. Then for any v in G, there exists a minimum dominating set for G that contains both v and b.

Proof. If v is not adjacent to b then let S be any set such that $[v,S] \rightarrow b$ then $S \cup \{v, b\}$ is a minimum dominating set that contains both v and b. Thus we need only consider the case where v is adjacent to b. But $\delta(b)=2$, so let u be the other vertex of G that is adjacent to b. Then v and u are not adjacent, so there exists a k-1 element set S containing v that dominates all of G-u. But then $S \cup \{b\}$ is a k-element dominating set for G that contains both v and b.♦

Lemma 6. Let G_1 be a k_1-good graph and G_2 a k_2-good graph with both $k_1, k_2 \geq 2$. Then the symmetric sum, G, of G_1 and G_2 is a $(k_1 + k_2)$- good graph having $\mathrm{diam}(G) = \mathrm{diam}(G_1) + \mathrm{diam}(G_2) + 1$.

Proof. Let a_1 and b_1 be diametric vertices of G_1 and a_2 and b_2 be diametric vertices of G_2. Form the symmetric sum, G, of G_1 and G_2 by expanding b_1 in G_1 and a_2 in G_2. Let $G_1'=<G_1,b_1,b_1'>$ and $G_2'=<G_2,a_2,a_2'>$ (see Figure 4).

Now clearly $\mathrm{diam}(G) = \mathrm{diam}(G_1) + \mathrm{diam}(G_2) + 1$, and a_1 and b_2 satisfy the conditions to be diametric vertices of G. Thus, we need only verify that G is k-domination symmetric where $k = k_1 + k_2$.

Note that :

(i). By Lemma 4, G_1' and G_2' are both domination-critical graphs.

(ii) If v and u are non-adjacent vertices of $G_1'-\{b_1, b_1'\}$, then both $v \to u$ and $u \to v$ in G_1'. (a similar assertion holds with respect to G_2.)

(iii). If $v \in G_1$, then there exists a set $S \subseteq G_1$ of cardinality k_1-1 such that S dominates all of $G_1 - \{v\}$.

(iv). $G_1' - b_1 \cong G_1$ is also k_1-good.

First we will show that $\gamma(G)=k$. Let S_1 be a minimum dominating set for G_1' and S_2 a minimum dominating set for G_2'. Then Clearly $S_1 \cup S_2$ is a k-element set that dominates G and so, $\gamma(G) \le k$. Now suppose that $\gamma(G) < k$, and let S be a dominating set for G with fewer than k vertices. Then without loss of generality, we may assume that $|S \cap G'_1| < k_1$. Hence $S \cap G'_1$ can not dominate all of G_1'. Now since S does dominate all of G, $S \cap G'_1$ must dominate all elements in $G - \{b_1, b_1'\}$ and thus must be of size $k_1 - 1$. Thus $S \cap G'_2$ must dominate both b_1 and b_1'. But this means that both a_2 and a_2' belong to $S \cap G'_2$. Hence $S \cap G'_2 - \{a_2'\}$ would be a k_2-1 element set that dominates all of G_2'.

With this contradiction it follows that $\gamma(G) = k$.

Now let v and u be non-adjacent vertices of G. We must show that $u \to v$ and $v \to u$. It is straightforward to check that all possible cases are equivalent to one of the following five.

Case 1. v and u both belong to $G_1' - \{b_1, b_1'\}$.

Let S_1 be a set of k_1-2 vertices of G_1' such that $S_1 \cup \{v\}$ dominates all of G_1' except for u. Let S_2 be any k_2-element dominating set for G_2'. Then $S_1 \cup S_2 \cup \{v\}$ dominates all of G except for u. Thus $v \rightarrow u$ in G. Similarly, $u \rightarrow v$ in G.

Case 2. $v \in G_1' - \{b_1, b_1'\}$, and $u \in G_2' - \{a_2, a_2'\}$.

Let S_1 be a k_1-1 element set in G_1' such that S_1 dominates all of G'_1 except for v. Let S_2 be any k_2-element dominating set for G_2' that contains u. Then $S_1 \cup S_2$ is a k-1 element set containing u that dominates all of G except for v. Thus $v \rightarrow u$ in G. Similarly, $u \rightarrow v$ in G.

Case 3. $v \in G_1' - \{b_1, b_1'\}$, and $u = a_2'$.

Let S_1 be any k_1-1 element subset of G_1' such that S_1 dominates all of G_1' except for v. Let S_2 be any k_2-element dominating set for G_2' that contains a_2'. Then $S_1 \cup S_2$ is a k-1 element set containing a_2' that dominates all of G except for v. Thus $u \rightarrow v$. As a consequence of Lemma 5 we can find a k_1-element dominating set S_1 for G_1' that contains b_1 and v. Let S_2 be any k_2-1 element set that dominates all of G_2' except for a_2 and a_2'. Then $S_1 \cup S_2$ is a k-1 element set containing v that dominates all of G except for a_2'. Thus, $v \rightarrow u$.

Case 4. $v = b_1$, and $u = a_2'$.

Let S_1 be any k_1- element subset of G_1' containing b_1 that dominates all of G_1'. Let S_2 be any k_2-1 element subset of G_2' that dominates all of G_2' except for a_2 and a_2'. Then $S_1 \cup S_2$ is a k-1 element set containing v that dominates all G except for a_2'. Thus $v \rightarrow u$. Similarly, $u \rightarrow v$.

Case 5. $v \in G_1' - \{b_1, b_1'\}$, and $u = b_1$.

It is straightforward to check that $b_1 \rightarrow v$. We will show that $v \rightarrow b_1$.

Let S_2 be a k_2-2 element set in G_2' such that $[a_2',S_2] \rightarrow b_2$. Let S_1 be a k_1-2 element set in G_1' such that $[v, S_1] \rightarrow b_1$ in G_1. Then $S= S_1 \cup S_2 \cup \{a_2', b_2\}$ is a k-2 element set such that $[v,S] \rightarrow b_1$ in G.

Thus G is k-domination-symmetric and the theorem follows.♦

Theorem 7. For every k≥2, there is a k-domination critical graph G having diameter $\left[\frac{3}{2}k-1\right]$. (Here [x] denotes the greatest integer less than or equal to x).

Proof. For $n \le 3$, we have the graphs of Figure 3. For $n \ge 4$, we define G_n to be the symmetric sum of G_{n-2} and G_2. Hence, for $n \ge 4$, $\mathrm{diam}(G_n) = \mathrm{diam}(G_{n-2}) + 3$. That the G_n's satisfy the condition of the Theorem is now an easy induction argument.♦

V. The Diameter of 4-critical Graphs

In this section we show that the bound of 2k-2 is not sharp for k-critical graphs with k = 4 by proving:

Theorem 8. The diameter of a 4-critical graph is at most 5.

For the remainder of this section, suppose that G is a 4-critical, connected graph with diameter 6. Let $W=\{V_0,V_1,\cdots, V_6\}$ be a level-decomposition of G, where $V_0=\{a\}$. Also, whenever we refer to a set S such that $[x,S] \rightarrow y$ for some vertices x and y, we will assume that S has cardinality 2.

Lemma 7. Any vertex of level V_5 that dominates all of level V_5 also dominates V_6.

Proof. Let x be a vertex of V_5 that dominates all of V_5, and let y be any vertex of level V_2. Suppose that x does not dominate all of V_6. Now x is not adjacent to y, and so there must exist S such that either $[x,S] \to y$ or $[y,S] \to x$. If $[x,S] \to y$, then since x does not dominate all of V_6, S must contain a vertex of $V_5 \cup V_6$, and in order to dominate a, S must also contain vertex of $V_0 \cup V_1$. But then, no element of $S \cup \{x\}$ is available to dominate the elements of V_3. So suppose that $[y,S] \to x$. Then in order to dominate a, S must contain a vertex of $V_0 \cup V_1$ and in order to dominate all of $V_4 \cup V_5 \cup V_6$ (except for x)S must also contain a vertex of V_5. But then this vertex would be adjacent to x which is impossible. ♦

Lemma 8. No vertex of V_3 dominates $V_2 \cup V_3$.

Proof. Suppose that x is a vertex of V_3 that dominates $V_2 \cup V_3$. Then there must exist a set S such that either $[a,S] \to x$ or $[x,S] \to a$ .If $[a,S] \to x$, then some vertex z of S must belong to V_1 in order to dominate V_2 but not x. On the other hand, if $[x,S] \to a$, then some vertex z of S must belong to V_2 in order to dominate V_1 but not a. In either case $\{a,x\}$ dominates N[z], and so $(S-\{z\}) \cup \{x,a\}$ would be a 3-element dominating set for G. Thus no such vertex x can exist.♦

Now we can prove Theorem 8.

Suppose that a, b are vertices of the connected k-critical graph G with diam(G) = d(a,b) = 6.

We may assume with no loss of generality that for some 2-element set S, $[a,S] \to b$. Let $W=\{V_0,V_1,\cdots, V_6\}$ be a level-decomposition of G with respect to a. Now in order for S to dominate level V_2, there must be exactly one vertex t of S in $V_4 \cup V_5 \cup V_6$. Now t must dominate all of V_5, and hence by Lemma 7, $t \notin V_5$.

Case 1. $t \in V_6$.

In this case the other vertex of S must dominate all of V_2, V_3 and V_4, which is impossible by Lemma 8.

Case 2. $t \in V_4$.

Recall that t dominates V_5 and note that in this case $V_6 = \{b\}$. Now there must exist a 2-element set X such that either $[a, X] \to t$ or $[t, X] \to a$.

Case 2.1 Suppose first that $[a, X] \to t$. Then by Lemma 1, t can not be a cut-vertex of G, and so V_4 contains more than just t itself. Now in order to dominate b without dominating t, it must be that $b \in X$. The remaining vertex x of X must dominate $V_2 \cup V_3$, and hence by Lemma 8, x must belong to V_2. But then there is no element of S that can dominate $V_4 - \{ t \}$.

Case 2.2 So suppose that $[t, X] \to a$. There is one vertex of X in $V_5 \cup V_6$, and so the other vertex x of X must dominate all of V_1 and V_2. Since x can not be adjacent to a, we have $x \in V_2$.

Now x is not adjacent to t, and so there exists a set Y such that either $[t, Y] \to x$ or $[x, Y] \to t$.

Suppose that $[t, Y] \to x$. Then x can not be a cut-vertex of G, and so V_2 must not be just $\{x\}$. But then since Y must contain the vertex a (in order not to dominate x) and some vertex of Y must be in $V_5 \cup V_6$, there is no vertex of Y available to dominate the other vertices of V_2.

So finally, suppose that $[x, Y] \to t$. Then t is not a cut-vertex, so V_4 contains vertices other than t. Then Y must contain a vertex of $V_0 \cup V_1$, as well as the vertex b, and there is no vertex available to dominate the other vertices of V_4. ♦

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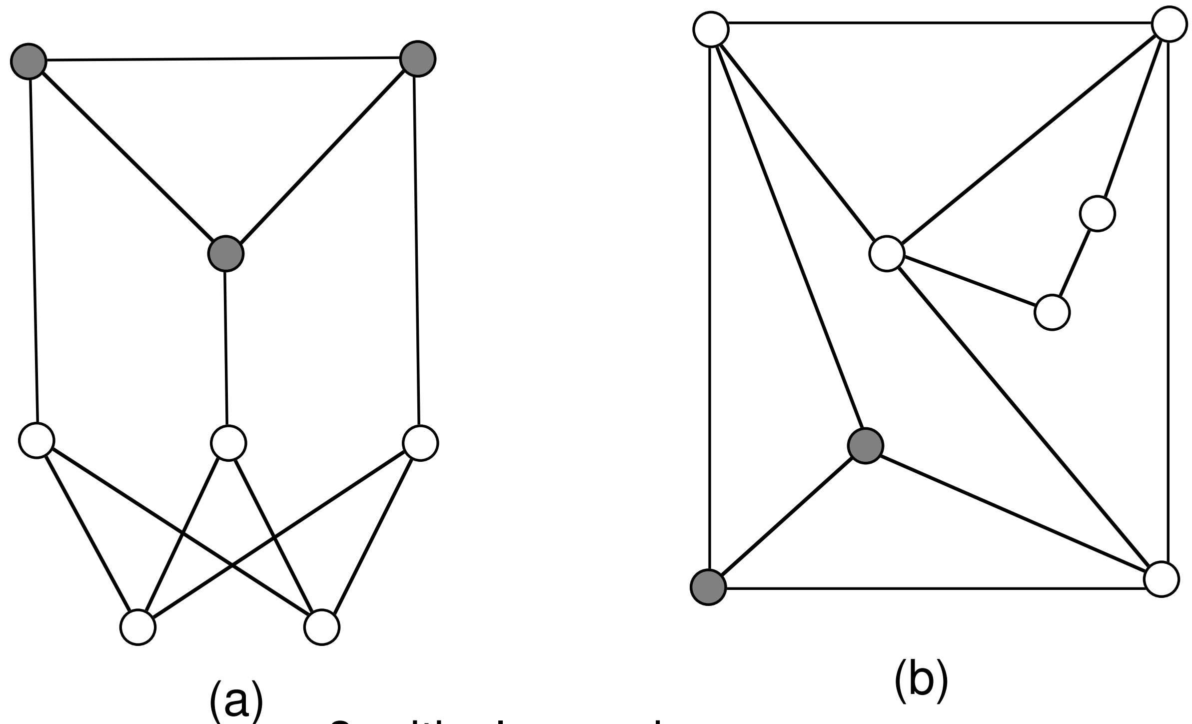


3-critical graphs

The darkened vertices represent the nucleus of the graph

Figure 1.

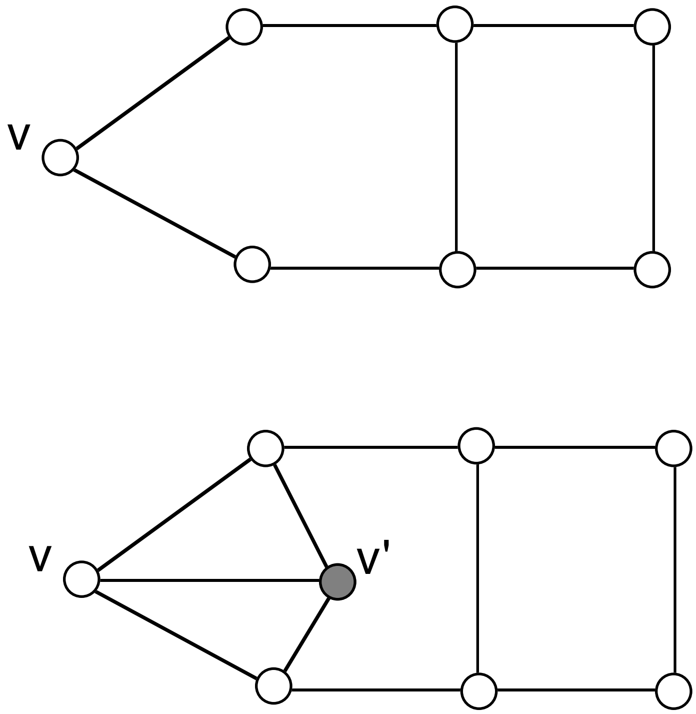


Figure 2

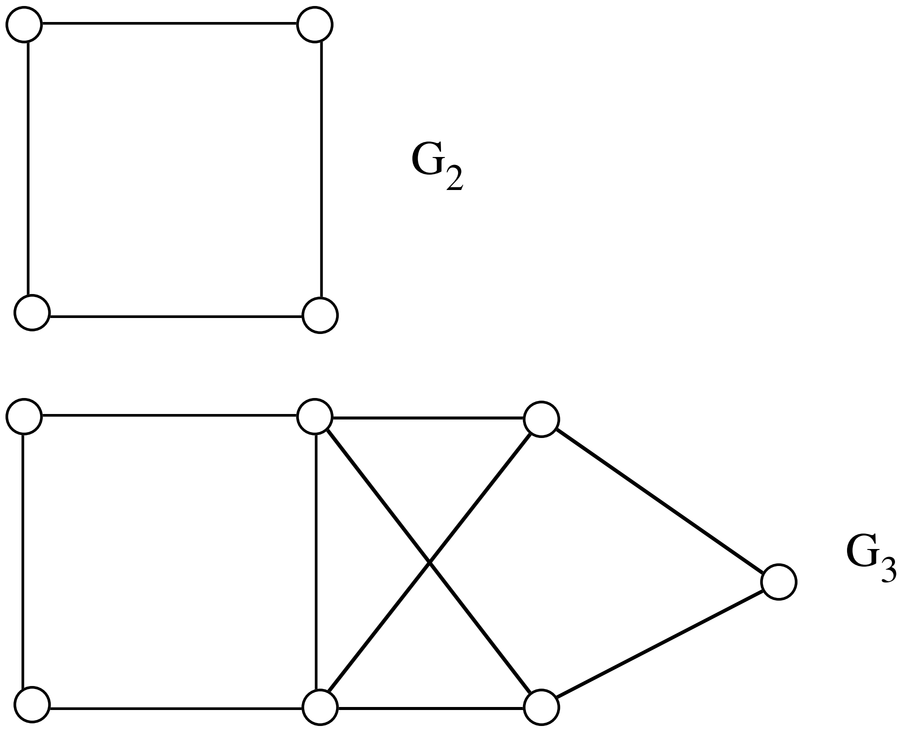
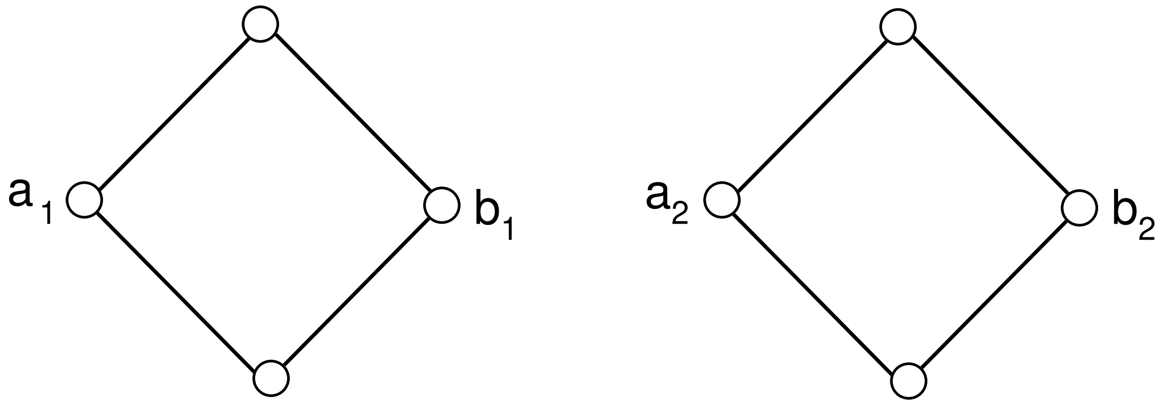


Figure 3



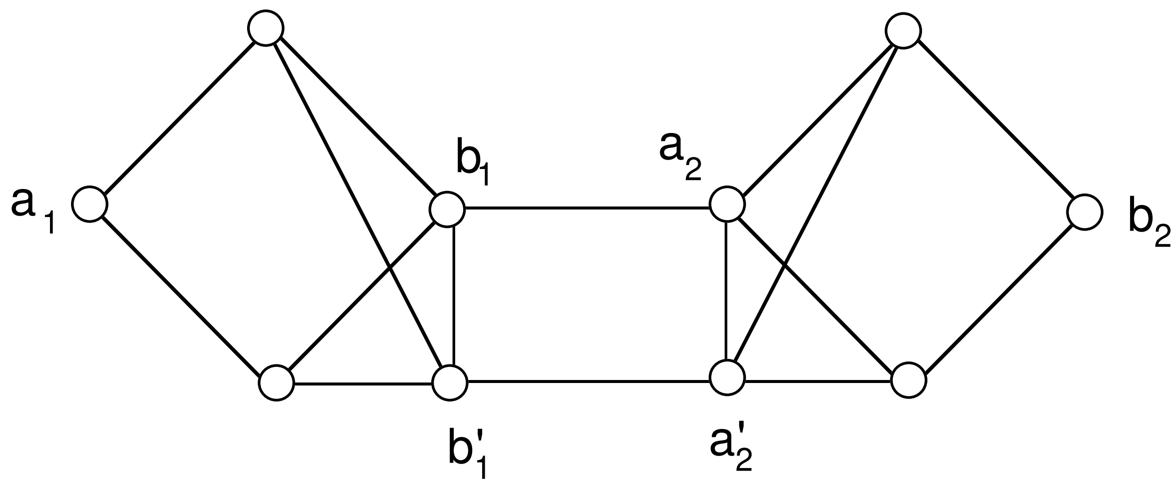


Figure 4