Homework 7.1 # 10, 17, 23, 29, 30

10) $\int \sin^{-1} x \, dx \quad = x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$\boxed{\begin{array}{l} u = \sin^{-1}x, \; dv = dx \\ du = \frac{1}{\sqrt{1-x^2}}\,dx, \; v = x \end{array}}$ $\quad\boxed{\begin{array}{l} t = 1-x^2 \\ dt = -2x\,dx \\ x\,dx = -\frac{1}{2}\,dt \end{array}}$

$= x\sin^{-1}x + \frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt$

$= x\sin^{-1}x + \frac{1}{2}\int t^{-1/2}\,dt = x\sin^{-1}x + \frac{1}{2}\cdot\frac{1}{1/2}t^{1/2}$

$= x\sin^{-1}x + \sqrt{1-x^2} + C.$

17) $\int e^{2\theta}\sin(3\theta)\,d\theta \leftarrow *$

$\boxed{\begin{array}{ll} u = e^{2\theta}, & dv = \sin(3\theta)\,d\theta \\ du = 2e^{2\theta}\,d\theta, & v = -\frac{1}{3}\cos(3\theta) \end{array}}$

$= -\frac{1}{3}\cos(3\theta)\,e^{2\theta} + \frac{2}{3}\int \cos(3\theta)\,e^{2\theta}\,d\theta$

$\boxed{\begin{array}{ll} t = e^{2\theta}, & dz = \cos(3\theta)\,d\theta \\ dt = 2e^{2\theta}\,d\theta, & z = \frac{1}{3}\sin(3\theta) \end{array}}$

$= -\frac{1}{3}\cos(3\theta)e^{2\theta} + \frac{1}{3}e^{2\theta}\sin(3\theta) - \frac{2}{3}\underbrace{\int \sin(3\theta)\,e^{2\theta}\,d\theta}_{*}.$ So

$\frac{5}{3}\int e^{2\theta}\sin(3\theta)\,d\theta = \frac{1}{3}e^{2\theta}\left(\sin(3\theta) - \cos(3\theta)\right) + C.$

Now multiply through by $\frac{3}{5}$ -

23) $\displaystyle\int_1^2 \frac{\ln x}{x^2}\,dx$

$$\boxed{\begin{array}{ll} u = \ln x & dv = x^{-2}\,dx \\ du = \frac{1}{x}\,dx & v = -\frac{1}{x} \end{array}}$$

$$= -\frac{1}{x}\ln x \Big|_{x=1}^{x=2} + \int_1^2 \frac{1}{x}\cdot\frac{1}{x}\,dx$$

$$= -\frac{1}{2}\ln 2 + 1\ln 1 + \left(-\frac{1}{x}\right)\Big|_{x=1}^{x=2}$$

$$= -\frac{1}{2}\ln 2 + \left(-\frac{1}{2}+1\right)$$

$$= \frac{1}{2} - \frac{1}{2}\ln 2 = \frac{1}{2}\left(1-\ln 2\right).$$

29) $\displaystyle\int \cos x \ln(\sin x)\,dx = \int \ln t\,dt$

$$\boxed{\begin{array}{l} t = \sin x \\ dt = \cos x\,dx \end{array}} \qquad \boxed{\begin{array}{ll} u = \ln t & dv = dt \\ du = \frac{1}{t}\,dt & v = t \end{array}}$$

$$= t\ln t - \int t\cdot\frac{1}{t}\,dt = t\ln t - t$$

$$= \sin x \ln(\sin x) - \sin x + C.$$

30) $$\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr = \frac{1}{2}\int_4^5 \frac{u-4}{\sqrt{u}}\,du$$

$$\boxed{\begin{array}{l} u = 4+r^2 \rightarrow u-4 = r^2 \\ du = 2r\,dr \end{array}}$$

$$= \frac{1}{2}\left(\int_4^5 u^{1/2}\,du - 4\int_4^5 u^{-1/2}\,du\right)$$

$$= \frac{1}{2}\left(\frac{2}{3}u^{3/2} - 4\cdot 2u^{1/2}\Big|_{u=4}^{u=5}\right)$$

$$= \frac{1}{2}\left(\frac{2}{3}5\sqrt{5} - 8\sqrt{5} - \left(\frac{2}{3}\cdot 8 - 16\right)\right)$$

$$= \frac{1}{2}\left(-\frac{14}{3}\sqrt{5} - \left(\frac{-3\cdot 16 + 16}{3}\right)\right)$$

$$= \frac{1}{2}\left(-\frac{14}{3}\sqrt{5} + \frac{16(3-1)}{3}\right)$$

$$= \frac{1}{2}\left(\frac{32 - 14\sqrt{5}}{3}\right) = \frac{16 - 7\sqrt{5}}{3}.$$