

MATH 554.01 - ANALYSIS I
SAMPLE TEST 3 – NOVEMBER 15, 2001

Name: _____

Directions: To receive credit, you must justify your statements unless otherwise stated. Answers should be provided in complete sentences.

1. a.) Define **open cover** for a set.
b.) Give an example of an open cover of a set A which has no finite subcover.
2. a.) Define what it means for a set to be **compact**.
b.) Suppose K is compact and $f : K \rightarrow \mathbb{R}$ is continuous. Prove that $f[K]$ is compact.
3. State and sketch a proof of the Heine-Borel theorem.
4. State and prove the **Intermediate Value Theorem**.
5. Prove that a compact subset of real numbers is bounded.
6. If A is a connected subset of real numbers, then show that A is an interval.
7. a.) Suppose that f is a continuous function on an interval I and changes sign, i.e. there exists x_1, x_2 in I so that $f(x_1)f(x_2) < 0$. Show that f has a zero in the interval, i.e. that there exists x_0 in I so that $f(x_0) = 0$.
b.) Prove or give an example to disprove the statement if the domain of f is not an interval.