Math 554/703I - Analysis I
Test 1 – September 20, 2001

1	(16 pts)
2	(10 pts)
3	(15 pts)
4	(24 pts)
5	(10 pts)
6	(15 pts)
7	(10 pts)

Name: ____________________________

Directions: To receive credit, you must justify your statements unless otherwise stated. Answers should be provided in complete sentences.

1. Pick exactly two of the following three parts to work: Suppose that F is an ordered field,

   (a) prove for each $a \in F$, $a \cdot 0 = 0$.

   (b) prove that $0 < 1$.

   (c) if $a < b$ and $c < d$, prove that $a + c < b + d$.

2. (a) Give a precise definition for a set to be finite.

(b) Give a precise definition for a set to be countably infinite.

3. Let A be an nonempty subset of $\mathbb{R}$.

a.) Define 'upper bound' for A.

b.) Define 'least upper bound' for A.

c.) Prove that least upper bounds are unique.

4. a. State and prove the Archimedean principle.

b. Prove that for each $\epsilon > 0$, there exists a natural number N such that for all $N \leq n$ there holds $0 < \frac{1}{n} < \epsilon$.

5. Negate the statement:

for each $\epsilon > 0$ there is a natural number N such that for every $n \geq N$ it is implied that $|a_n - a| < \epsilon$

6. For $a > 0$, and all natural numbers n, prove that

$$1 + na < (1 + a)^n$$

7. Prove that if $0 < r < 1$ and $\epsilon > 0$, then there exists a natural number n so that $r^n < \epsilon$. (Hint: Problem #6)