Math 554 - #3

(Fill in the proofs from Lectures.)

Theorem. Suppose that $\lim_{n\to\infty} a_n = a$, then prove that $\lim_{n\to\infty} |a_n| = |a|$.

Theorem. Convergent sequences are bounded.

Defn. A sequence $\{a_n\}$ is called *monotone increasing* if $a_m \le a_n$ whenever $m \le n$. A sequence $\{a_n\}$ is called *monotone decreasing* if $a_n \le a_m$ whenever $m \le n$.

Theorem. Monotone sequences, which are also bounded, converge.

Theorem. Suppose that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = a$. If $a_n \le c_n \le b_n$ for all $n \in I\!N$, then $\lim_{n\to\infty} c_n$ exists and equals a.

Theorem. (Properties of Limits) Suppose that $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then

1. $\lim_{n\to\infty} a_n + b_n = a + b$
2. $\lim_{n\to\infty} a_n b_n = ab$
3. If $b \neq 0$, then $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{a}{b}$.

Defn. A sequence $\{a_n\}$ is called *Cauchy* if for each $\epsilon > 0$ there is an $N \in I\!N$ so that $|a_m - a_n| < \epsilon$ whenever $m, n \ge N$.

Theorem. Each convergent sequence is Cauchy.

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