# Math 524
## Dichotomous Search

**Example.** This is an illustration of how the Dichotomous search algorithm performs on the function $f(x) = x^2$ over the interval $[-5, 15]$. We have chosen $\epsilon = 10^{-3}$, which means that we will do approximately 14 iterations which will consequently involve 28 function evaluations. Since our function is $f(x) = x^2$, it is easy to compare the values of $f(a_{new})$ and $f(b_{new})$.

| $[\ a$ | $a_{new}$ | $b_{new}$ | $b\ ]$ |
|---|---|---|---|
| -5.0000e+00 | 5.0000e+00 | 5.0010e+00 | 1.5000e+01 |
| -5.0000e+00 | 5.0000e-04 | 1.5000e-03 | 5.0010e+00 |
| -5.0000e+00 | -2.4992e+00 | -2.4983e+00 | 1.5000e-03 |
| -2.4992e+00 | -1.2489e+00 | -1.2479e+00 | 1.5000e-03 |
| -1.2489e+00 | -6.2369e-01 | -6.2269e-01 | 1.5000e-03 |
| -6.2369e-01 | -3.1109e-01 | -3.1009e-01 | 1.5000e-03 |
| -3.1109e-01 | -1.5480e-01 | -1.5380e-01 | 1.5000e-03 |
| -1.5480e-01 | -7.6648e-02 | -7.5648e-02 | 1.5000e-03 |
| -7.6648e-02 | -3.7574e-02 | -3.6574e-02 | 1.5000e-03 |
| -3.7574e-02 | -1.8037e-02 | -1.7037e-02 | 1.5000e-03 |
| -1.8037e-02 | -8.2686e-03 | -7.2686e-03 | 1.5000e-03 |
| -8.2686e-03 | -3.3843e-03 | -2.3843e-03 | 1.5000e-03 |
| -3.3843e-03 | -9.4214e-04 | 5.7861e-05 | 1.5000e-03 |
| -9.4214e-04 | 2.7893e-04 | 1.2789e-03 | 1.5000e-03 |

The following table gives the progression of bracketed intervals for the minimizer and an upper bound for the minimal value of $f$ over that interval.

| $k$ | $a_k$ | $b_k$ | $f_k$ |
|---|---|---|---|
| 1 | -5.0000e+00 | 5.0010e+00 | 2.5000e+01 |
| 2 | -5.0000e+00 | 1.5000e-03 | 2.5000e-07 |
| 3 | -2.4992e+00 | 1.5000e-03 | 6.2413e+00 |
| 5 | -1.2489e+00 | 1.5000e-03 | 1.5572e+00 |
| 6 | -6.2369e-01 | 1.5000e-03 | 3.8774e-01 |
| 7 | -3.1109e-01 | 1.5000e-03 | 9.6158e-02 |
| 8 | -1.5480e-01 | 1.5000e-03 | 2.3653e-02 |
| 9 | -7.6648e-02 | 1.5000e-03 | 5.7227e-03 |
| 10 | -3.7574e-02 | 1.5000e-03 | 1.3377e-03 |
| 11 | -1.8037e-02 | 1.5000e-03 | 2.9026e-04 |
| 12 | -8.2686e-03 | 1.5000e-03 | 5.2832e-05 |
| 13 | -3.3843e-03 | 1.5000e-03 | 5.6848e-06 |
| 14 | -9.4214e-04 | 1.5000e-03 | 3.3479e-09 |
| 15 | -9.4214e-04 | 1.2789e-03 | 7.7802e-08 |