Math/Stat 511

Key – HW #3

p.82

#17 Ordered permutation of 3 types of objects (A, B, & C) with # A = 5, # B = 6, # C = 7. $\binom{18}{5,6,7} = \frac{18!}{5!6!7!} = \boxed{14{,}702{,}688}$

#18 (a) 19 white out of 52 Hearts. Select 9 at random without replacement, unordered.
Probability (exactly 3 white) $= \frac{\binom{19}{3}\binom{33}{6}}{\binom{52}{9}} \doteq \boxed{.2917}$

(b) $P(\text{3 white, 2 tan, 1 pink, 1 yellow, 2 green}) = \frac{\binom{19}{3}\binom{10}{2}\binom{7}{1}\binom{3}{0}\binom{5}{1}\binom{2}{0}\binom{6}{2}}{\binom{52}{9}}$

---

p.92 **#1** (a) $P(B_1) = \frac{5\times10^3}{1\times10^6} = \boxed{5\times10^{-3}}$ (b) $P(A_1) = \frac{78{,}515}{10^6} \doteq \boxed{7.85\times10^{-2}}$

(c) $P(A_1|B_2) = \frac{73{,}630}{995{,}000} \doteq \boxed{.0800}$ (d) $P(B_1|A_1) = \frac{4885}{78{,}515} \doteq \boxed{.0622}$

(e) probability of a 'false positive' is ≈ 8%, ≈ 6.22% chance of AIDS if test is positive.

#4 (a) A = {heart on 1st draw}, B = {heart on second draw}
$P(A\cap B) = P(B|A)\cdot P(A) = \frac{12}{51}\cdot\frac{13}{52} \doteq \boxed{.0588}$

(b) C = {club on second draw} ⇒ $P(A\cap C) = P(C|A)\,P(A) = \frac{13}{51}\,\frac{13}{52} \doteq \boxed{.0637}$

(c) D = {ace on second draw}, A_1 = {ace of hearts on 1st draw}, A_2 = {heart but not ace on 1st draw}
$A_1 \cup A_2 = A$, using the hint...
$P(A\cap D) = P(A_1\cap D) + P(A_2\cap D) = P(D|A_1)P(A_1) + P(D|A_2)\cdot P(A_2)$
$= \frac{3}{51}\,\frac{1}{52} + \frac{4}{51}\,\frac{12}{52} \doteq \boxed{.0192}$

#7 (a) {(R,W), (R,R), (W,R), (W,W)}, (b) A = {(R,R)}, B = {(R,R), (R,W), (W,R)}
$P(A|B) = \frac{P(A\cap B)}{P(B)} = \boxed{\frac{1}{3}}$

#11 3 defectives out of 12. Selected in order w/o replacement

(a) E = {3rd defective is 3rd tested} $P(E) = \frac{3}{12}\,\frac{2}{11}\,\frac{1}{10} = \boxed{\frac{1}{220}}$

(b) E = {5th bulb tested is defective}
F = {2 of the 1st 4 are defective}
$P(E|F) = \frac{1}{8}$, (1 defective & 7 good left)
$P(F) = \frac{\binom{3}{2}\binom{9}{2}}{\binom{12}{4}} = \frac{12}{55}$

P(5th tested is the 3rd defective) $= P(E|F)\cdot P(F)$
$= \boxed{\frac{3}{110}}$

(c) P(10th tested is 3rd defective) $= \frac{1}{3}\cdot\frac{\binom{3}{2}\binom{9}{7}}{\binom{12}{9}} = \boxed{9/55}$

#17 4 Black, 6 Brown, 8 Olive ; 2 selected @ random w/o replacement

(a) P[both same color] = P[both Black] + P[both Brown] + P[both Olive]

mutually exclusive event

$$P[\text{both Black}] = P[2^{nd}\text{ Black} \mid 1^{st}\text{ Black}] \cdot P[1^{st}\text{ Black}]$$

$$= \frac{3}{17} \cdot \frac{4}{18} = \frac{12}{306}$$

$$P[\text{both Brown}] = \frac{5}{17} \cdot \frac{6}{18} = \frac{30}{306} \quad , \quad P[\text{both Olive}] = \frac{7}{17} \cdot \frac{8}{18} = \frac{56}{306}$$

$$\Longrightarrow \quad P[\text{both same color}] = \frac{98}{306} = \boxed{\frac{49}{153}}$$

(b) $$P[\underset{A}{\text{both olive}} \mid \underset{B}{\text{both same color}}] = \frac{P(A \cap B)}{P(B)} = \frac{\left(\frac{56}{306}\right)}{\left(\frac{98}{306}\right)} = \boxed{\frac{4}{7}}$$