Differential Equations
(Math 242.501)
Sample Questions for Test #3 – May 8, 2002

Name: ____________________________

1	(25 *pts*)
2	(30 *pts*)
3	(20 *pts*)
4	(10 *pts*)
5	(15 *pts*)

Directions:

Answer all questions in the space provided. You can also use the back of the facing opposite page if you need more room. You must show intermediate work for partial credit. Calculators are allowed.

1. Compute the Laplace transform of the following functions:

   a.) $t^2 \sin(2t)$

   b.) $f(t) := \begin{cases} \cos(2t), & \pi/4 \le t \le \pi/2; \\ 0, & \text{otherwise.} \end{cases}$ by using the unit step functions.

   c.) $\tan(t)\ \delta_\pi$

2. Compute the following inverse Laplace transforms:

   a.) $\dfrac{s}{(s+1)^2}$

   b.) $\dfrac{s\ e^{-\frac{\pi s}{2}}}{s^2+4}$

   c.) $\dfrac{1}{s^2(s-1)^2}$

3. Compute the **Laplace transform of the solution** to each of the following equations:

   a.) $y'' - y' = e^t \cos(t), \quad y(0) = 1, y'(0) = -1.$

   b.) $y'(t) = 1 - \sin(t) - \int_0^t y(\tau)\, d\tau, \quad$ y(0)=0.

4. Determine the 2-nd order Taylor approximation to $f(x) = x \ln x$ about $x = 1$. Provide an error estimate over the interval $(1, 1.1)$.

5. Use step size $h = .1$ and apply the Runge-Kutta method of second order, with parameter $b = 1$, to estimate $y(.2)$ where y is the solution to the equation $y' = (x-y)^3, \quad y(0) = .5$.