Differential Equations
(Math 242.501)
Sample Test 2 – April 2, 2002

Directions:

Answer all questions in the space provided. You can also use the back of the facing opposite page if you need more room. You must show intermediate work for partial credit. Calculators are allowed.

1. Consider the differential equation $y'' + 2y' + 5y = 0$.
   a.) Determine the auxiliary equation.
   b.) Compute the associated Wronskian.
   c.) Determine the general solution for the equation.

2. Consider the differential equation $y'' - 4y' + 3y = 2e^{2t}$.
   a.) Determine the homogeneous solution.
   b.) Compute a particular solution.
   c.) Determine the general solution for the equation.
   d.) Find the solution to the initial value problem when $y(1) = 0$ and $y'(1) = 1$.

3. Find the form of a particular solution to the equation
$$y''' - 2y'' + y' = xe^x + 5.$$

4. Compute the general solution for
$$4x^2y'' + y = 0$$

5. Use the variation of parameters method to determine a particular solution for the equation
$$x^2y'' + xy' + y = \sec(\ln x)$$

6. Determine (do not solve) a mass-spring-dashpot system that would be modelled by the equation
$$4y'' + 5y' + 3y = t\sin(2t)$$
$$y(0) = 2, y'(0) = 0$$

7. Use the definition of the Laplace transform to compute $L[f](s)$ where
$$f(t) = \begin{cases} 1, & 0 < t \leq 1; \\ e^{-t}, & 1 < t. \end{cases}$$

Compute the Laplace transform of each of the following:

8. $f(t) = (t-2)^2e^{3t+5}$
9. $f(t) = 2t^3 - 4\sin(4t)$
10. $f(t) = \cos(2t)\cosh(t)$