DIFFERENTIAL EQUATIONS
(MATH 242.501)
SAMPLE TEST 1 – FEBRUARY 14, 2002

Name: ____________________________

Directions: Answer all questions in the space provided. You can also use the back of the facing opposite page if you need more room. Calculators are allowed, but you must show intermediate work for partial credit.

Preliminaries and Theoretical Background

1. a.) State precisely the theorem (hypothesis and conclusion) for existence and uniqueness of a first order initial value problem.

   b.) Consider the equation $y' + x^2 y = e^{x^3}$ with initial conditions $y(x_0) = y_0$. Briefly discuss if this has a solution, if it is unique and why.

2. Verify that the function $y(x) = \sin(\ln x)$ satisfies the initial value problem

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = x^3$$
$$y(0) = 0, \quad \frac{dy}{dx}(0) = 1$$

Modelling with first order ODE

3. A tank contains 200 liters of fluid in which 30 grams of salt is dissolved. Brine containing 1 gram of salt per liter is then pumped into the tank at a rate of 4 liters/min; the well-mixed solution is pumped out at the same rate. Find $A(t)$ the number of grams of salt in the tank at time t.

Classification/Closed Form Solution Methods:
(Separation of Variables,Linear inhomogeneous/multiplying factor, Exact Equations)

In each of the following three problems, first classify the equation and then solve.

4. Solve for the general solution of

$$xy' + 4y = x^3 - x$$

.

5. Solve for the general solution of

$$y' + \frac{3x^2y + e^y}{x^3 + xe^y - 2y} = 0$$

6. a. Solve for the general solution of

$$y' - e^{3x+2y} = 0$$

   b. Find a particular solution if y(0)=2.

Qualitative Methods
(Slope field, Phase Line and asymptotes)

7. Consider the equation $y' = 10 + 3y - y^2$.
   a.) Find the critical points and phase line of the equation.
   b.) Classify each critical point as asymptotically stable, unstable, or semi-stable.
   c.) Sketch the slope field and a couple of representative solutions.

Quantitative Methods: Euler's method

8. Use Euler's method with step size $h = .1$ to obtain a 4 decimal place approximation to $y(.3)$ where y is the solution to the initial value problem

$$\begin{aligned} &y' = (x - y)^2 \\ &y(0) = \tfrac{1}{2}. \end{aligned}$$