

## Comparison of Numerical Methods

**Example:**  $y' = (x+y-1)^2$ ,  $y(0)=2$ . Intermediate computations for the Improved Euler method.

$x_j$	$F(x_j, y_j)$	$y_{j+1}^*$	$F(x_{j+1}, y_{j+1}^*)$	$y_j$
0	0	2	0	2
0.1	1	2.1	1.44	2.122
0.2	1.4933	2.2713	2.1648	2.3049
0.3	2.2647	2.5314	3.3539	2.5858
0.4	3.5564	2.9415	5.4825	3.0378

### Example of Euler vrs. Improved Euler & 4-th order Runge-Kutta

$$y' = 2xy, \quad y(1) = 1$$

x	y(x)	<u>Euler Error</u>				<u>Improved Euler Error</u>				<u>Runge-Kutta 4<sup>th</sup> order</u>			
		h=.1	h=.01	h=.001	h=.0001	h=.1	h=.01	h=.001	h=.0001	h=.1	h=.01	h=.001	h=.0001
1.0	1.00000	0.00000	0.00000	0.00000	0.00000	0.0000	0.0000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
1.1	1.23370	0.03370	0.00390	0.00040	0.00000	0.0017	0.0000	1.91E-07	1.91E-09	0.00E+00	4.00E-10	1.00E-14	2.90E-14
1.2	1.55270	0.08870	0.01040	0.00110	0.00010	0.0048	0.0001	5.55E-07	5.56E-09	0.00E+00	1.40E-09	6.00E-14	7.80E-14
1.3	1.99370	0.17840	0.02140	0.00220	0.00020	0.0106	0.0001	1.23E-06	1.23E-08	0.00E+00	3.30E-09	1.90E-13	1.56E-13
1.4	2.61170	0.32430	0.03980	0.00410	0.00040	0.0209	0.0002	2.47E-06	2.47E-08	1.00E-04	7.40E-09	4.80E-13	2.85E-13
1.5	3.49030	0.56250	0.07060	0.00730	0.00070	0.0394	0.0005	4.72E-06	4.73E-08	1.00E-04	1.58E-08	1.13E-12	4.88E-13
1.6	4.75880	0.95270	0.12270	0.01260	0.00130	0.0725	0.0009	8.79E-06	8.81E-08	3.00E-04	3.27E-08	2.52E-12	8.30E-13
1.7	6.61940	1.59520	0.21120	0.02180	0.00220	0.1316	0.0016	1.62E-05	1.62E-07	5.00E-04	6.65E-08	5.42E-12	1.41E-12
1.8	9.39330	2.66100	0.36270	0.03760	0.00380	0.2378	0.0029	2.97E-05	2.97E-07	1.10E-03	1.35E-07	1.14E-11	2.33E-12
1.9	13.59910	4.44310	0.62470	0.06510	0.00650	0.4297	0.0053	5.44E-05	5.45E-07	2.10E-03	2.71E-07	2.39E-11	3.96E-12
2.0	20.08550	7.45030	1.08250	0.11320	0.01140	0.7792	0.0098	1.00E-04	1.00E-06	4.30E-03	5.47E-07	4.94E-11	6.72E-12
<b># of (rel.) flops</b>		10	100	1,000	10,000	20	200	2,000	20,000	40	400	4,000	40,000

*Be sure to observe rates of convergence, # of arithmetic operations, and finally degradation due to roundoff error.*