MATH 142 - CALCULUS II (SECTIONS 11-12)
TEST 2 – MARCH 3, 2005

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	(104*pts*)

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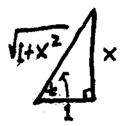
Directions: No Calculators are allowed on Tests or Exams. To receive parital credit, you must show your work and *box* your final answer. **On each problem, state the method you plan to use.**

If a problem involves a trigonometric substitution which reduces to a trigonometric integral of a standard type, then you may leave your answer in that integral form.

1. Simplify $\sin(\arctan x)$.

$t = \arctan x \Leftrightarrow \tan t = x$,



and so

$$\sin(\arctan x) = \sin t = \boxed{\frac{x}{\sqrt{1+x^2}}}$$

Determine the following antiderivatives:

2. $\int \sqrt{x}\ln x\,dx$ Integration by parts

$$\text{Integral} = \int \underbrace{\ln x}_{u}\ \underbrace{\sqrt{x}\,dx}_{dv} = \int \ln x\ d\left[\tfrac{2}{3}x^{3/2}\right] = \tfrac{2}{3}x^{3/2}\ln x - \int \tfrac{2}{3}x^{3/2}\,d[\ln x]$$

$$= \tfrac{2}{3}x^{3/2}\ln x - \tfrac{2}{3}\int x^{3/2}\tfrac{1}{x}\,dx$$

$$= \tfrac{2}{3}x^{3/2}\ln x - \tfrac{2}{3}\,\tfrac{2}{3}x^{3/2} + c$$

$$= \boxed{\tfrac{2}{3}x^{3/2}\ln x - \tfrac{4}{9}x^{3/2} + c}$$

3. $\int \sin x\cos^2 2x\,dx$ Identity $(\cos 2x = 2\cos^2 x - 1)$ & Trigonometric integral

$$\text{Integral} = \int (2\cos^2 x - 1)^2 \sin x\,dx \overset{\substack{u=\cos x\\ du/dx=-\sin x}}{=} \int (2u^2-1)^2(-1)\,du = -\left(\int (4u^4 - 4u^2 + 1)\,du\right)$$

odd power of sine

$$= -\tfrac{4}{5}u^5 + \tfrac{4}{3}u^3 - u + c$$

$$= \boxed{-\tfrac{4}{5}\cos^5 x + \tfrac{4}{3}\cos^3 x - \cos x + c}$$

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4. $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$ trig substitution: $x = 2\sin t$

Integral $= \int \frac{4\sin^2 t}{\sqrt{4(1-\sin^2 t)}}(2\cos t\,dt) = \int \frac{4\sin^2 t}{2\sqrt{\cos^2 t}}\,2\cos t\,dt$

$= \boxed{4\int \sin^2 t\,dt} = 4\int \frac{1-\cos 2t}{2}\,dt = 2\int (1-\cos 2t)\,dt$

full credit

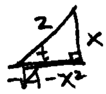
$= 2t - \sin(2t) + c$

$= 2t - 2\sin t\cos t + \cancel{c}$

$= \boxed{2\arcsin(\tfrac{x}{2}) - x\cdot\frac{\sqrt{4-x^2}}{2} + c}$ + 5 pts. extra credit

$t = \arcsin(\tfrac{x}{2})$

$\cos t = \frac{\sqrt{4-x^2}}{2}$ since



5. $\int \frac{2x-3}{x^3-3x^2+2x}\,dx$ Partial Fractions

$x^3 - 3x^2 + 2x = x(x-1)(x-2) \Rightarrow \frac{2x-3}{x^3-3x^2+2x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$

Solving for A, B, & C gives: $\frac{2x-3}{x^3-3x^2+2x} = \frac{(-3/2)}{x} + \frac{(1)}{x-1} + \frac{(\frac{1}{2})}{x-2}$

Integral $= -\tfrac{3}{2}\int \frac{dx}{x} + \int \frac{dx}{x-1} + \frac{1}{2}\int \frac{dx}{x-2}$

$= \boxed{-\tfrac{3}{2}\ln|x| + \ln|x-1| + \tfrac{1}{2}\ln|x-2| + c}$

6. $\int x\arctan(x)\,dx$ Integration by parts

Integral $= \int \arctan(x)\,d[\tfrac{1}{2}x^2] = \tfrac{1}{2}x^2\arctan x - \int \tfrac{1}{2}x^2\,d[\arctan x]$

$= \tfrac{1}{2}x^2\arctan x - \tfrac{1}{2}\int \frac{x^2}{1+x^2}\,dx$

But $\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$ by polynomial division & so

Integral $= \boxed{\tfrac{1}{2}x^2\arctan x - \tfrac{1}{2}(x - \arctan x) + c}$

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7. $\int e^{2x} \cos(x)\, dx$ Integration by Parts

+/-	d	$\int$
+	e^{2x}	$\cos x$
−	$2e^{2x}$	$\sin x$
+	$4e^{2x}$	$-\cos x$

$$\text{Integral} =: I = e^{2x}\sin x + 2e^{2x}\cos x - 4\cdot I$$

$$\Longrightarrow \quad 5\cdot I = e^{2x}\sin x + 2e^{2x}\cos x$$

$$\boxed{\text{Integral} = \frac{1}{5}\left(e^{2x}\sin x + 2e^{2x}\cos x\right)}$$

8. $\int \sin^2 x \cos^2 x\, dx$ Trigonometric Integral

①
$$\text{Integral} = \int \frac{(2\sin x \cos x)^2}{4}dx = \frac{1}{4}\int \sin^2 2x\, dx \overset{u=2x}{=} \frac{1}{8}\int \sin^2 u\, du$$

$$= \frac{1}{8}\left(\frac{u}{2} - \frac{\sin 2u}{4}\right) + c$$

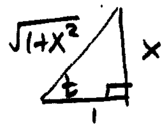
$$= \boxed{\frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + c}$$

or

②
$$\text{Integral} = \int \sin^2 x(1-\sin^2 x)dx = \int \sin^2 x\, dx - \int \sin^4 x\, dx = A + B$$

A done earlier & solve B using integration by parts.

9. $\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$ Trig substitution : $x = \tan t$



$$\text{Integral} = \int \frac{\sec^2 t\, dt}{(1+\tan^2 t)^{3/2}} = \int \frac{\sec^2 t}{\sec^3 t}dt$$

$$= \boxed{\int \cos t\, dt} = \sin t + c = \boxed{\frac{x}{\sqrt{1+x^2}} + c}$$

full credit to get this trigon. integral

+5 pts extra credit

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10. $\int \frac{x^2+1}{x^2-x}\,dx$ Partial Fractions

$$\frac{x^2+1}{x^2-x} = \frac{(x^2-x)+(x+1)}{x^2-x} = 1 + \frac{x+1}{x(x-1)} = 1 + \frac{A}{x} + \frac{B}{x-1}$$

But then $x+1 = A(x-1) + Bx$ & so $A=-1$, $B=2$

$$\text{Integral} = \int\left(1 + \frac{(-1)}{x} + \frac{2}{x-1}\right)dx$$

$$= \boxed{x - \ln|x| + 2\ln|x-1| + c}$$

11. $\int \sin 3x \cos x\,dx$ trig identity

$$\sin 3x \cos x = \frac{\sin 4x + \sin 2x}{2}$$

$$\text{Integral} = \int\left(\tfrac{1}{2}\sin 4x + \tfrac{1}{2}\sin 2x\right)dx$$

$$= \boxed{-\tfrac{1}{8}\cos 4x - \tfrac{1}{8}\cos 2x + c}$$

12. $\int \frac{3x}{\sqrt{2x-x^2}}\,dx$ Complete the square & trig substitution

$$\text{Integral} = \int \frac{3x}{\sqrt{1-(x-1)^2}}\,dx \underset{u=x-1}{=} \int \frac{3(u+1)}{\sqrt{1-u^2}}\,du = 3\left(\int\frac{u}{\sqrt{1-u^2}}\,du + \int\frac{du}{\sqrt{1-u^2}}\right)$$

$$\text{But } \int\frac{u}{\sqrt{1-u^2}}\,du \underset{v=1-u^2,\ \frac{dv}{du}=-2u}{=} -\tfrac{1}{2}\int v^{-1/2}\,dv = -v^{1/2}+c = -\sqrt{1-u^2}+c$$

$$= -\sqrt{2x-x^2}+c$$

$$\int\frac{du}{\sqrt{1-u^2}} = \arcsin u + c = \arcsin(x-1) + c$$

$$\Rightarrow \text{Integral} = \boxed{3\left(-\sqrt{2x-x^2} + \arcsin(x-1)\right) + c}$$

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13. $\int \frac{x+1}{x^2+2x+5}\,dx$ complete square & u-substitution: $u = x+1$

$$\text{Integral} = \int \frac{x+1}{(x+1)^2+4}\,dx = \underset{u=x+1}{\int \frac{u}{u^2+4}\,du} = \underset{v=u^2+4}{\frac{1}{2}\int \frac{dv}{v}}$$

$$= \frac{1}{2}\ln|v| + c = \frac{1}{2}\ln|u^2+4| + c$$

$$= \boxed{\frac{1}{2}\ln|x^2+2x+5| + c}$$