MATH 142 - CALCULUS II (SECTIONS 11-12)
TEST 1 – FEBRUARY 3, 2005

1	(10 *pts*)
2	(10 *pts*)
3	(30 *pts*)
4	(10 *pts*)
5	(15 *pts*)
6	(15 *pts*)
7	(10 *pts*)

Name: ______________________

Directions: No Calculators are allowed on Tests or Exams. To receive proper credit, you must show your work and *box* your final answer.

1. Compute the derivative of $F(x) := \int_{\sqrt{x}}^{2x} \sqrt{1+t^2}\, dt$

Let $G(x) = \int_0^x \sqrt{1+t^2}\,dt$, then by the Fundamental Theorem of Calculus $G'(x) = \sqrt{1+x^2}$. $F(x) = G(2x) - G(\sqrt{x})$

so application of the chain rule gives

$$F'(x) = G'(2x)\cdot(2x)' - G'(\sqrt{x})\cdot(\sqrt{x})'$$
$$= \left(\sqrt{1+(2x)^2}\right)(2) - \left(\sqrt{1+(\sqrt{x})^2}\right)\left(\tfrac{1}{2}x^{-1/2}\right)$$
$$= \boxed{2\sqrt{1+4x^2} - \tfrac{1}{2}x^{-1/2}\sqrt{1+x}}$$

2. (a) Express $(x+\sin x)^{\frac{1}{x}}$ in terms of the ln and exp functions.

$$(x+\sin x)^{\frac{1}{x}} = e^{\ln\left((x+\sin x)^{1/x}\right)}$$
$$= \boxed{e^{\frac{1}{x}\ln(x+\sin x)}}$$

(b) Differentiate $(x+\sin x)^{\frac{1}{x}}$.

$$\left(e^{\frac{1}{x}\cdot\ln(x+\sin x)}\right)' = e^{\frac{1}{x}\ln(x+\sin x)}\cdot\left(\tfrac{1}{x}\ln(x+\sin x)\right)'$$
$$= \boxed{e^{\frac{1}{x}\ln(x+\sin x)}\left[-1\cdot x^{-2}\ln(x+\sin x) + x^{-1}\left(\frac{1+\cos x}{x+\sin x}\right)\right]}$$

Name: ____________________________

3. Determine each of the following integrals:

(a) $\int (3x-2)^4\,dx$

Let $u = 3x-2$, then $\frac{du}{dx} = 3$.

$$\int (3x-2)^4\,dx = \int u^4 \left(\tfrac{1}{3}\right) du = \frac{1}{15} u^5 + c \Big|_{u=3x-2}$$

$$= \boxed{\frac{1}{15}(3x-2)^5 + c}$$

(b) $\int x\sqrt{1-x}\,dx$

Let $u = 1-x$, then $\frac{du}{dx} = -1$. Also $x = 1-u$.

$$\int x\sqrt{1-x}\,dx = \int (1-u)\,u^{\frac{1}{2}}(-1)\,du$$

$$= \int (u-1)\,u^{\frac{1}{2}}\,du = \int \left(u^{3/2} - u^{\frac{1}{2}}\right) du$$

$$= \left(\tfrac{2}{5} u^{5/2} - \tfrac{2}{3} u^{3/2} + c\right)\Big|_{u=1-x}$$

$$= \boxed{\tfrac{2}{5}(1-x)^{5/2} - \tfrac{2}{3}(1-x)^{3/2} + c}$$

(c) $\int_0^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Let $u = \sqrt{x}$, then $\frac{du}{dx} = \frac{1}{2}\frac{1}{\sqrt{x}}$.

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int (\sin u)(2)\,du = (-2\cos u + c)\Big|_{u=\sqrt{x}}$$

$$= -2\cos(\sqrt{x}) + c.$$

Therefore

$$\int_0^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = -2\cos(\sqrt{x})\Big|_{x=0}^{x=\pi^2} = -2(\cos\pi - 1)$$

$$= \boxed{4}$$

Name: ______________________________

(d) $\int \cos^2(2x)\sin(2x)\,dx$

Let $u = \cos 2x$, then $\frac{du}{dx} = (-\sin 2x)\cdot 2$.

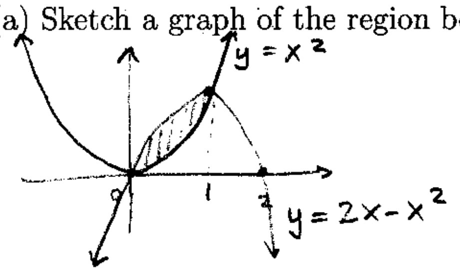
$$\int \cos^2(2x)\sin(2x)\,dx = \int u^2\left(-\tfrac{1}{2}\right)du$$
$$= \left(-\tfrac{1}{6}u^3 + c\right)\Big|_{u=\cos 2x}$$
$$= \boxed{-\tfrac{1}{6}(\cos 2x)^3 + c}$$

(e) $\int \frac{2x+(x+5)^{\frac{1}{3}}}{x+5}\,dx$

Let $u = x+5$, then $\frac{du}{dx} = 1$ & $x = u-5$.

$$\int \frac{2x+(x+5)^{1/3}}{x+5}dx = \int \frac{2(u-5)+u^{1/3}}{u}du = \int\left(2 - 5\tfrac{1}{u} + u^{-2/3}\right)du$$
$$= \left(2u - 5\ln u + 3u^{1/3} + c\right)\Big|_{u=x+5} = \boxed{\begin{array}{c}2(x+5) - 5\ln(x+5)\\ + 3(x+5)^{1/3} + c\end{array}}$$

4. (a) Sketch a graph of the region bounded by the curves $y = x^2$ and $y = 2x - x^2$.



intersection points: $x^2 = 2x - x^2$

or $2x^2 - 2x = 0$

$x(x-1) = 0$

$\boxed{x = 0, 1}$

(b) Determine the area of the specified region.

$$A = \int_0^1 \left[(2x - x^2) - x^2\right]dx = \left(x^2 - \tfrac{2}{3}x^3\right)\Big|_{x=0}^{x=1} = \boxed{\tfrac{1}{3}}$$

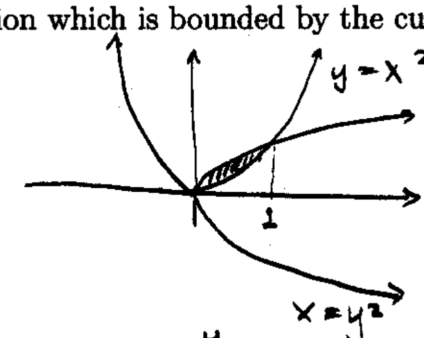
Name: ______________________

5. Using the *Disk/Washer* method, determine the volume of revolution, about the x-axis, of the region bounded by the curves $y = x^2$ and $y = 2x - x^2$.

Using problem # 4,

$$\text{Volume} = \int_0^1 \pi\left[(2x-x^2)^2 - (x^2)^2\right]dx$$
$$= \int_0^1 \pi(4x^2 - 4x^3 + x^4 - x^4)\,dx$$
$$= \pi\left(\tfrac{4}{3}x^3 - x^4\right)\Big|_{x=0}^{x=1} = \boxed{\tfrac{\pi}{3}}$$

6. Using the *Cylindrical Shell* method, determine the volume of revolution about the y-axis of the region which is bounded by the curves $x = y^2$ and $y = x^2$. Sketch the volume.

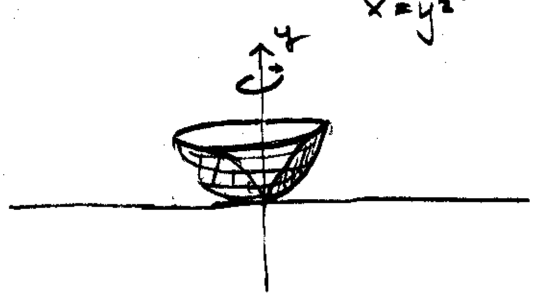


Intersection pts:

$$\left.\begin{array}{l} y = x^2 \\ y = \sqrt{x} \end{array}\right\} \Rightarrow x^2 = \sqrt{x}$$
or $x^4 - x = 0$

$$\Rightarrow x(x^3 - 1) = 0$$

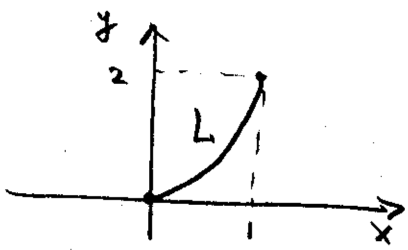
$$\boxed{x = 0, 1}$$



$$V = \int_0^1 2\pi x(\sqrt{x} - x^2)\,dx$$
$$= 2\pi \int_0^1 (x^{3/2} - x^3)\,dx$$
$$= 2\pi\left(\tfrac{2}{5}x^{5/2} - \tfrac{1}{4}x^4\right)\Big|_{x=0}^{x=1}$$
$$= 2\pi\left(\tfrac{3}{5} - \tfrac{1}{4}\right) = \boxed{\tfrac{3}{10}\pi}$$

Name: ____________________

7. Determine the arclength of the curve given by the graph of $y = 2x^{\frac{3}{2}}$ for $x = 0$ to $x = 1$.



$$f(x) = 2x^{3/2}$$

$$L = \int_0^1 \sqrt{1 + [f'(x)]^2}\,dx$$

$$= \int_0^1 \sqrt{1 + (3\sqrt{x})^2}\,dx$$

$$= \int_0^1 \sqrt{1 + 9x}\,dx$$

To evaluate this integral, let $u = 1 + 9x$, then $\frac{du}{dx} = 9$.

$$L = \int_1^{10} u^{1/2} \left(\tfrac{1}{9}\right) du$$

since $u = 1$ when $x = 0$ & $u = 10$ when $x = 1$.

$$= \frac{1}{9}\frac{2}{3} u^{3/2} \Big|_{u=1}^{u=10}$$

$$= \boxed{2/27\,(10^{3/2} - 1)}$$