Final Exam

Math 141.1

Dec. 17, 1993

Directions: Show your work for full credit. Clearly indicate in your work the problem number and your answer.

1. Determine if the following limits exist and if so their value:
   (a) $\lim_{x\to 1} \dfrac{x^2-3x+2}{x^2-1}$
   (b) $\lim_{x\to \infty} \dfrac{3x^2+sin(x)}{1-x^2}$
   (c) $\lim_{x\to 0} \dfrac{cos(x)-1}{x}$
   (d) $\lim_{x\to 3^-} \dfrac{1}{x^2-9}$
2. Using the definition of *derivative* and properties of limits, compute $f'(1)$ where $f(x) := 2+x-x^2$.
3. Determine derivatives for each of the following functions:
   (a) $f(x) = (x^2-1)$,
   (b) $\sqrt{3x^3-4x+1}$
   (c) $\sqrt{sin(3x^2)}$
4. Using differentials, estimate $cos(.05)$.
5. If the width of a rectangle is changing at a rate of twice its length and its length is changing at a rate of +5 cm/sec, then at what rate is the area of the rectangle changing?
6. Consider the function
$$f(x) = x^3 - x^2 - x + 2, \qquad -2 \le x \le 4.$$
   (a) Determine the intervals where f is increasing and where it decreases.
   (b) Determine the critical points for the function.
   (c) Determine the intervals where f is concave up and where it is concave down.
   (d) Specify which points are local maxima, local minima, or inflection points.
   (e) Sketch a graph of f using the information determined above.
7. Evaluate each of the antiderivatives:
   (a) $(2x^2-3)(x^2+2x)$
   (b) $x\sqrt{3-x^2}$
   (c) $tan(x)sec(x)$
8. Calculate the following definite integrals:

(a) $\int_0^2 (x^2-1)^2 dx$

(b) $\int_0^1 x\sqrt{3-x^2}dx$

9. Compute the area bounded by the curves $y = x^2 - 2$ and $2x^2 + x - 4$.

10. Compute the volume of revolution about the axis of the curve

$$y = x^2 - 2x, \quad 1 \leq x \leq 2.$$