

Solutions for HW 5

Pg. 312: 3 Solution:

Assume first that E is Lebesgue measurable and let A be given. Then there exists a measurable set $G \supset A$ such that $m^*(A) = m(G)$. Then $m^*(A) = m(G) = m(G \cap E) + m(G \cap E^c) \geq m^*(A \cap E) + m^*(A \cap E^c)$. By subadditivity we always have $m^*(A) \leq m^*(A \cap E) + m^*(A \cap E^c)$, so E is Caratheodory measurable. Assume now that E is Caratheodory measurable. Let $E_n = E \cap \overline{B(0; n)}$. Then there exist for each n a measurable set $G_n \supset E_n$ such that $m(G_n) = m^*(E_n)$. We can assume that $G_n \subset \overline{B(0; n)}$. Then $G_n \cap E = G_n \cap E \cap \overline{B(0; n)} = G_n \cap E_n = E_n$. Taking now $A = G_n$ in the Caratheodory measurability property of E we find $m(G_n) = m^*(G_n \cap E) + m^*(G_n \setminus E) = m^*(E_n) + m^*(G_n \setminus E)$. Hence $m^*(G_n \setminus E) = 0$, as $m(G_n) < \infty$. Let $G = \cup_n G_n$. Then G measurable, $G \supset E$, and $m^*(G \setminus E) = 0$. Hence $G \setminus E$ is measurable and thus $E = G \setminus (G \setminus E)$ is measurable.

Pg. 91:9 Solution From $\alpha \chi_{E_\alpha} \leq f$ it follows that $\alpha m(E_\alpha) \leq \int f dx$.