

MATH 141 EXAMPLES

Show all work for full credit.

1. Suppose a particle moves along an s -axis with velocity

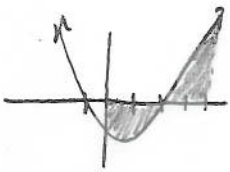
$$v(t) = t^2 - t - 2 \text{ meters/min.}$$

- a. Find the displacement (change in position) of the particle over the first 4 minutes.

$$\begin{aligned} \text{Displacement} &= \int_0^4 (t^2 - t - 2) dt \\ &= \left[\frac{t^3}{3} - \frac{t^2}{2} - 2t \right]_0^4 \\ &= \left(\frac{64}{3} - 8 - 8 \right) - 0 = \frac{64}{3} - \frac{48}{3} = \boxed{\frac{16}{3} \text{ meters}} \end{aligned}$$

- b. Find the total distance traveled by the particle in the first 4 minutes.

$$\begin{aligned} t^2 - t - 2 &= 0 \\ (t-2)(t+1) &= 0 \\ t &= 2, -1 \end{aligned}$$



$$\begin{aligned} \int_0^2 (t^2 - t - 2) dt &= \left[\frac{t^3}{3} - \frac{t^2}{2} - 2t \right]_0^2 \\ &= \left(\frac{8}{3} - 2 - 4 \right) - 0 \\ &= \frac{8}{3} - \frac{18}{3} = -\frac{10}{3} \end{aligned}$$

$$\begin{aligned} \int_2^4 (t^2 - t - 2) dt &= \left[\frac{t^3}{3} - \frac{t^2}{2} - 2t \right]_2^4 \\ &= \left(\frac{64}{3} - 8 - 8 \right) - \left(\frac{8}{3} - 2 - 4 \right) \\ &= \frac{16}{3} - \left(-\frac{10}{3} \right) = \frac{26}{3} \end{aligned}$$

$$\text{Total Distance} = \frac{10}{3} + \frac{26}{3} = \frac{36}{3} = \boxed{12 \text{ meters}}$$

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2. A ball is thrown upward with a speed of 72 ft/sec from the edge of a cliff 40 ft above the ground. Assuming free-fall motion, what is the maximum height of the ball?

$$v_0 = 72 \quad s(t) = -16t^2 + 72t + 40$$

$$s_0 = 40$$

$$v(t) = -32t + 72$$

$$-32t + 72 = 0$$

$$\frac{-32t}{-32} = \frac{-72}{-32} = \frac{9}{4} \text{ seconds}$$

$$s\left(\frac{9}{4}\right) = -16\left(\frac{81}{16}\right) + 72\left(\frac{9}{4}\right) + 40$$

$$= -81 + 162 + 40$$

$$= \boxed{121 \text{ ft}}$$

$$\begin{array}{l} \leftarrow \oplus \quad \ominus \rightarrow v(t) \\ \rightarrow \frac{9}{4} \downarrow s(t) \end{array}$$

3. A stone is thrown downward from the top of a 160 ft high cliff with an initial speed of 48 ft/sec. Assuming free-fall motion, what is the speed of the stone upon impact with the ground?

$$s_0 = 160$$

$$s(t) = -16t^2 - 48t + 160$$

$$v(t) = -32t - 48$$

$$v_0 = -48$$

$$-16t^2 - 48t + 160 = 0$$

$$v(2) = -32(2) - 48$$

$$-16(t^2 + 3t - 10) = 0$$

$$v(2) = -112$$

$$-16(t+5)(t-2) = 0$$

$$t = -5 \quad t = 2$$

$$\text{Speed} = \boxed{112 \text{ ft/sec}}$$

4. A rock, dropped from an unknown height, strikes the ground with a speed of 24 ft/sec. Assuming free-fall motion, find the height from which the rock was dropped.

$$s_0 = h$$

$$s(t) = -16t^2 + 0t + h$$

$$v(t) = -32t$$

$$v_0 = 0$$

$$s(t) = -16t^2 + h$$

$$\frac{-32t}{-32} = \frac{-24}{-32}$$

$$-16\left(\frac{3}{4}\right)^2 + h = 0$$

$$t = \frac{3}{4}$$

$$-9 + h = 0$$

$$h = \boxed{9 \text{ ft}}$$