SOLUTIONS

MATH 111 PRETEST 1

This test is designed to give an example of what types of questions may be on the test. **Show all work for full credit.**

1. Rewrite each of the following using interval notation.

a. $x \leq -3$ or $x > 7$

$(-\infty, -3] \cup (7, \infty)$

b. $x > 2$ and $x \leq 8$

$(2, 8]$

c. x is greater than 7 but at most 12

$7 < x \leq 12$ $\quad (7, 12]$

d. x is at least 3

$x \geq 3$ $\quad [3, \infty)$

2. Perform the operation and write your answer as a polynomial in standard form.

a. $5x - 2(x - 3)$

$= 5x - 2x + 6$

$= 3x + 6$

b. $(2x + 3)^2 - (x - 1)(x^2 - 2x + 3)$

$(2x+3)(2x+3) - (x^3 - 2x^2 + 3x - x^2 + 2x - 3)$

$= 4x^2 + 6x + 6x + 9 - x^3 + 2x^2 - 3x + x^2 - 2x + 3$

$= -x^3 + 7x^2 + 7x + 12$

c. $(2x + 5)(x - 7)(3x + 1)$

$= (2x+5)(3x^2 + x - 21x - 7)$

$= 6x^3 + 2x^2 - 42x^2 - 14x + 15x^2 + 5x - 105x - 35$

$= 6x^3 - 25x^2 - 114x - 35$

d. $x - 5[4 - 7(x + 8)]$

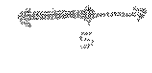
$= x - 5(4 - 7x - 56)$

$= x - 20 + 35x + 280$

$= 36x + 260$

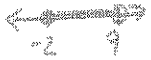
3. Graph the set.

a. $(-\infty, 5] \cap [-2, 9)$



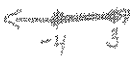
$= [-2, 5]$

b. $(-\infty, 5] \cup [-2, 9)$



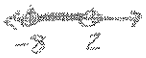
$= (-\infty, 9)$

c. $[-4, 3] \cap (-8, 2)$



$= [-4, 2)$

d. $[-4, 3] \cup (-8, 2)$



$= (-8, 3]$

4. Simplify each of the following. Express answers with positive exponents only.

a. $\frac{-35x^2y^4}{5x^6y^{-8}}$

$= -7x^{-4}y^{12}$

$= \boxed{\frac{-7y^{12}}{x^4}}$

b. $(4x^2)^3(2x^5)^{-5}$

$= 4^3x^6 2^{-5}x^{-25} = \frac{4^3x^{-19}}{2^5}$

$= \frac{64}{32x^{19}}$

$= \boxed{\frac{2}{x^{19}}}$

c. $\left(\frac{-30a^{14}b^8}{10a^{17}b^{-2}}\right)^{-3}$

$= \left(-3a^{-3}b^{10}\right)^{-3}$

$= (-3)^{-3}a^9b^{-30}$

$= \boxed{\frac{-a^9}{27b^{30}}}$

d. $\frac{(2x^2y^{-1})^{-3}(4xy^{-3})^2}{(3x^{-4}y)^{-2}}$

$= \frac{2^{-3}x^{-6}y^3 4^2 x^2 y^{-6}}{3^{-2}x^8 y^{-2}} = \frac{9 \cdot 16 x^{-4} y^{-3}}{8x^8 y^{-2}} = 18x^{-12}y^{-1} = \boxed{\frac{18}{x^{12}y}}$

5. Simplify the expression and express the answer using rational exponents.

a. $\sqrt[3]{8x^2}\sqrt{16x^3}$

$= 2x^{2/3} \cdot 4x^{3/2}$

$= \boxed{8x^{19/6}}$

$\frac{2}{3} + \frac{3}{2} = \frac{4}{6} + \frac{15}{6} = \frac{19}{6}$

b. $\frac{xy}{\sqrt[3]{27xy^2}}$

$= \frac{xy}{3x^{1/3}y^{2/3}}$

$= \boxed{\frac{1}{3}x^{2/3}y^{1/3}}$

$1 - \frac{1}{3} = \frac{3}{3} - \frac{1}{3} = \frac{2}{3}$

$1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} = \frac{1}{3}$

c. $\sqrt{x\sqrt[5]{x^3}}$

$= \left(x \cdot x^{3/5}\right)^{1/2}$

$= \left(x^{8/5}\right)^{1/2}$

$= \boxed{x^{4/5}}$

$1 + \frac{3}{5} = \frac{5}{5} + \frac{3}{5} = \frac{8}{5}$

$\frac{4}{8} \cdot \frac{1}{2} = \frac{4}{5}$

6. Find the exact value for each of the following without using your calculator.

a. 4^{-2}

$= \frac{1}{4^2} = \boxed{\frac{1}{16}}$

b. $16^{1/4}$

$= \sqrt[4]{16}$

$= \boxed{2}$

c. $64^{-3/2}$

$= \frac{1}{64^{3/2}}$

$= \frac{1}{\left(64^{1/2}\right)^3} = \frac{1}{8^3}$

$= \boxed{\frac{1}{512}}$

$\sqrt[3]{512}^2$

d. $(-27)^{2/3}$

$= \left((-27)^{1/3}\right)^2$

$= (-3)^2$

$= \boxed{9}$

7. Factor completely.

a. $3x^4 - 9x^3 - 30x^2$

$= 3x^2(x^2 - 3x - 10)$

$= \boxed{3x^2(x-5)(x+2)}$

b. $27x^3 - 125$

$= (3x)^3 - (5)^3$

$= \boxed{(3x-5)(9x^2+15x+25)}$

c. $6x^2 - 11x - 21$

$= \boxed{(6x+7)(x-3)}$

$7 \cdot 3$

$3 \cdot 7$

$1 \cdot 21$

$21 \cdot 1$

$(3x \quad)(2x \quad)$

$7 \quad 3$

$3 \quad 7$

$1 \quad 21$

$21 \quad 1$

d. $x^4 + x^3 + x + 1$

$= x^3(x+1) + 1(x+1)$

$= (x+1)(x^3+1)$

$= \boxed{(x+1)(x+1)(x^2-x+1)}$

e. $12x^2 + 28x - 24$

$= 4(3x^2 + 7x - 6)$

$= \boxed{4(3x-2)(x+3)}$

f. $3x^3 - 3x^2 - 12x + 12$

$= 3[x^3 - x^2 - 4x + 4]$

$= 3[x^2(x-1) - 4(x-1)]$

$= 3(x-1)(x^2-4) = \boxed{3(x-1)(x+2)(x-2)}$

8. Simplify the rational expression.

$$\frac{12+x-x^2}{2x^2-9x+4}$$

$$= \frac{-1(x^2-x-12)}{(2x-1)(x-4)}$$

$$= \frac{-1(x-4)(x+3)}{(2x-1)(x-4)}$$

$$= \boxed{\frac{-1(x+3)}{2x-1}}$$

9. Perform the multiplication and simplify. Give your answer in factored form.

$$\frac{2x^2+x-6}{x^2+4x-5} \cdot \frac{x^3-3x^2+2x}{4x^2-6x}$$

$$= \frac{(2x-3)(x+2)}{(x+5)(x-1)} \cdot \frac{x(x^2-3x+2)}{2x(2x-3)}$$

$$= \frac{(x+2)(x-2)(x-1)}{(x+5)(x-1)\cdot 2}$$

$$= \boxed{\frac{(x+2)(x-2)}{2(x+5)}}$$

10. Perform the division and simplify. Give your answer in factored form.

$$\frac{x^3-8}{x^2-4} \div \frac{x^2+2x+4}{x^3+8} \cdot \frac{x^3+8}{x^2+2x+4}$$

$$= \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)} \cdot \frac{(x+2)(x^2-2x+4)}{(x^2+2x+4)}$$

$$= \boxed{x^2-2x+4}$$

11. Perform the subtraction and simplify. Give your answer in factored form.

$$\frac{3}{x-1} - 5 \cdot \frac{(x-1)}{(x-1)} \qquad \text{LCD: } x-1$$

$$= \frac{3-5x+5}{x-1} = \boxed{\frac{8-5x}{x-1}}$$

12. Perform the addition and simplify. Give your answer in factored form.

$$\frac{2}{x^2-x-2} + \frac{10}{x^2+2x-8} \qquad \text{LCD: } (x-2)(x+1)(x+4)$$

$$\frac{(x+4)}{(x+4)} \cdot \frac{2}{(x-2)(x+1)} + \frac{10}{(x+4)(x-2)} \cdot \frac{(x+1)}{(x+1)}$$

$$= \frac{2x+8+10x+10}{(x+4)(x-2)(x+1)}$$

$$= \frac{12x+18}{(x+4)(x-2)(x+1)} = \boxed{\frac{6(2x+3)}{(x+4)(x-2)(x+1)}}$$

13. Perform the addition or subtraction and simplify. Give your answer in factored form.

$$-\frac{1}{x} + \frac{2}{x^2+1} + \frac{1}{x^3+x} \qquad \text{LCD: } x(x^2+1)$$

$$-\frac{1}{x}\cdot\frac{(x^2+1)}{(x^2+1)} + \frac{2}{x^2+1}\cdot\frac{x}{x} + \frac{1}{x(x^2+1)}$$

$$= \frac{-x^2-1+2x+1}{x(x^2+1)} = \frac{-x^2+2x}{x(x^2+1)} = \frac{-x(x-2)}{x(x^2+1)} = \boxed{\frac{-(x-2)}{x^2+1}}$$

14. Rationalize the denominator. Simplify your answer.

a. $\frac{6}{\sqrt{6}+2\sqrt{3}}$ · $\frac{\sqrt{6}-2\sqrt{3}}{\sqrt{6}-2\sqrt{3}}$

$= \frac{6(\sqrt{6}-2\sqrt{3})}{6-2\sqrt{18}+2\sqrt{18}-4\cdot 3}$

$= \frac{6(\sqrt{6}-2\sqrt{3})}{-6}$

$= \boxed{-\sqrt{6}+2\sqrt{3}}$

b. $\frac{3+\sqrt{6}}{2-\sqrt{6}}$ · $\frac{2+\sqrt{6}}{2+\sqrt{6}}$

$= \frac{6+3\sqrt{6}+2\sqrt{6}+6}{4+2\sqrt{6}-2\sqrt{6}-6}$

$= \boxed{\frac{12+5\sqrt{6}}{-2}}$

15. Solve for R.

$$x = \sqrt{\frac{Ry}{4q}}$$

$4q \cdot x^2 = \frac{Ry}{4q} \cdot 4q$

$\frac{4qx^2}{y} = \frac{Ry}{y}$

$\boxed{R = \frac{4qx^2}{y}}$

16. Solve.

$$\frac{12}{x^2-2x-8} - \frac{2}{x-4} = \frac{7}{x+2}$$

$(x-4)(x+2)$

LCD: $(x-4)(x+2)$

$$\frac{12(x-4)(x+2)}{(x-4)(x+2)} - \frac{2(x-4)(x+2)}{(x-4)} = \frac{7(x-4)(x+2)}{(x+2)}$$

$$12-2x-4 = 7x-28$$

$$8-2x = 7x-28$$

$$-9x = -36 \rightarrow \boxed{x=4}$$ extraneous → No Solution

17. Solve.

$$\frac{2}{x-3} - \frac{8}{x^2-9} = \frac{4}{x+3}$$

$(x+3)(x-3)$

LCD: $(x+3)(x-3)$

$$\frac{2(x+3)(x-3)}{x-3} - \frac{8(x+3)(x-3)}{(x+3)(x-3)} = \frac{4(x+3)(x-3)}{x+3}$$

$$2x+6-8 = 4x-12$$

$$2x-2 = 4x-12$$

$$-2x = -10$$

$$\boxed{x=5}$$ ✓

18. Solve.

$$\frac{24}{x^2-4x-12} + \frac{3x-1}{x-6} = \frac{2x+3}{x+2}$$

$(x-6)(x+2)$

LCD: $(x-6)(x+2)$

$$\frac{24(x-6)(x+2)}{(x-6)(x+2)} + \frac{(3x-1)(x-6)(x+2)}{(x-6)} = \frac{(2x+3)(x-6)(x+2)}{(x+2)}$$

$$24+3x^2+6x-x-2 = 2x^2-12x+3x-18$$

$$3x^2+5x+22 = 2x^2-9x-18$$

$$x^2+14x+40 = 0$$

$$(x+10)(x+4) = 0$$

$x+10=0$

$$\boxed{x=-10}$$ ✓

$x+4=0$

$$\boxed{x=-4}$$ ✓

19. Solve each of the following equations by factoring.

a. $x^3 + 3x^2 - x - 3 = 0$

$x^2(x+3) - 1(x+3) = 0$

$(x+3)(x^2-1) = 0$

$(x+3)(x+1)(x-1) = 0$

$x+3=0$ $\boxed{x=-3}$ $\quad x+1=0$ $\boxed{x=-1}$ $\quad x-1=0$ $\boxed{x=1}$

b. $4x^2 - 6x - 10 = x^2 - 9x + 8$

$3x^2 + 3x - 18 = 0$

$3(x^2 + x - 6) = 0$

$3(x+3)(x-2) = 0$

$3 \neq 0$ $\quad x+3=0$ $\boxed{x=-3}$ $\quad x-2=0$ $\boxed{x=2}$

20. Solve the following power equation. Simplify.

a. $(x+13)^2 - 25 = 0$

$(x+13)^2 = 25$

$x+13 = \pm 5$

$x = -13 \pm 5$

$x = -13-5 = \boxed{-18}$ $\qquad x = -13+5 = \boxed{-8}$

21. Use the Quadratic Formula to solve. Give the exact answer and simplify.

a. $x^2 + 14x + 44 = 0$

$a=1$ $\quad b=14$ $\quad c=44$

$$\frac{-14 \pm \sqrt{(14)^2 - 4(1)(44)}}{2(1)}$$

$$= \frac{-14 \pm \sqrt{20}}{2}$$

$$= \frac{-14 \pm 2\sqrt{5}}{2} = \boxed{-7 \pm \sqrt{5}}$$

b. $25x^2 - 20x + 3 = 0$

$a=25$ $\quad b=-20$ $\quad c=3$

$$\frac{20 \pm \sqrt{(-20)^2 - 4(25)(3)}}{2(25)}$$

$$= \frac{20 \pm \sqrt{100}}{50} = \frac{20 \pm 10}{50}$$

$\frac{20+10}{50} = \boxed{\frac{3}{5}}$ $\qquad \frac{20-10}{50} = \boxed{\frac{1}{5}}$

22. Solve the following by completing the square. Give the exact answer.

$$x^2 + 8x + 14 = 0$$

$x^2 + 8x = -14$

$x^2 + 8x + \underline{16} = -14 + \underline{16}$

$(x+4)^2 = 2$

$x + 4 = \pm\sqrt{2}$

$\boxed{x = -4 \pm \sqrt{2}}$

23. Solve the following by completing the square. Give the exact answer.

$$\frac{2x^2}{2} + \frac{8x}{2} - \frac{18}{2} = \frac{0}{2}$$

$x^2 + 4x - 9 = 0$

$x^2 + 4x + \underline{4} = 9 + \underline{4}$

$(x+2)^2 = 13$

$x + 2 = \pm\sqrt{13}$

$\boxed{x = -2 \pm \sqrt{13}}$

24. Solve the inequality and give your answer in interval notation.

$$2\left(\frac{1}{2}(8x+1) \geq 3x + \frac{5}{2}\right)2$$

$8x + 1 \geq 6x + 5$

$\frac{2x}{2} \geq \frac{4}{2}$

$x \geq 2$ $\quad \boxed{[2, \infty)}$

25. Solve the inequality and give your answer in interval notation.

$$-1 < 2 - \frac{x}{3} < 1$$

$$-2 \quad -2 \quad -2$$

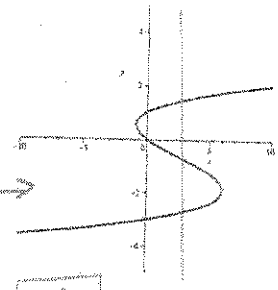
$$-3\left(-3 < -\frac{x}{3} < -1\right)-3$$

$$9 > x > 3$$

$$3 < x < 9 \quad \boxed{(3,9)}$$

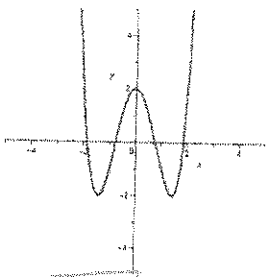
26. Decide whether each of the following could be the graph of a function.

Both pass the Vertical Line Test

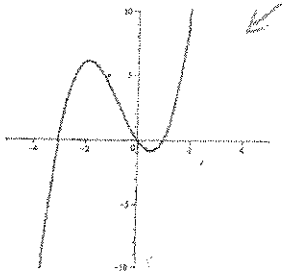


Does not pass the Vertical Line Test →

NO



Yes



Yes

27. In each of the following cases, decide which of the following statements is true:

A. y is a function of x
B. x is a function of y
C. both A and B
D. None of the above.

Explain your answers.

a.

x	-9	-8	-7	-5	-4
y	14	8	0	-9	13

Every input x has one output y
→ y is a function of x.

[C]

Every input y has one output x
→ x is a function of y.

b.

x	-4	1	-6	-7	1
y	6	1	5	10	4

An input of $x=1$ has two outputs $y=1$ and $y=4$.
→ y is not a function of x.

[B]

Every input y has one output x.
→ x is a function of y.

For each of the equations below, solve for the indicated variable. Simplify your answers. Your final answer should not have complex rational expressions.

28. Solve for s.

$$f\left(P = L + \frac{sx}{f}\right)f$$

$$fP = fL + sx$$

$$\frac{fP - fL}{x} = \frac{sx}{x}$$

$$\boxed{s = \frac{fP - fL}{x}}$$

29. Solve for f.

$$f\left(P = L + \frac{sx}{f}\right)f$$

$$fP = fL + sx$$

$$fP - fL = sx$$

$$\frac{f(P-L)}{P-L} = \frac{sx}{P-L} \longrightarrow \boxed{f = \frac{sx}{P-L}}$$

30. Solve for d.

$$L = A + (n-1)d$$

$$\frac{L-A}{n-1} = \frac{(n-1)d}{n-1}$$

$$\boxed{d = \frac{L-A}{n-1}}$$

31. Solve for n.

$$n\left(L = A + (n-2)\frac{180}{n}\right)n$$

$$nL = An + (n-2)180$$

$$nL = An + 180n - 360$$

$$360 = An + 180n - nL$$

$$\frac{360}{A+180-L} = \frac{n(A+180-L)}{A+180-L} \longrightarrow \boxed{n = \frac{360}{A+180-L}}$$