Hamilton-Jacobi equations and constrained L_1 minimization

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L_1-based minimization methods for first order nonlinear PDEs will be considered. In the case of stationary Hamilton-Jacobi equations

$$H(x, u, Du) = 0, \;\; x \in \Omega, \quad \text{with} \quad u|_{\partial\Omega} = 0$$

a convergence theory and numerical algorithms will be presented. We construct an approximate solution to this problem using a special type of constrained minimization. In the case of a convex and uniformly continuous hamiltonian, we will show that the sequence of approximate minimizers converges to the unique viscosity solution u of the problem. The main features of our methods are that they are of arbitrary polynomial order and do not have any artificial viscosity. We will discuss many open problems and the connection between this minimization process and classical constrained approximation.