Math 554
February 19, 2008

Name ______________________________________

[2 points apiece]
1. Let A and B be sets. Define the following:

   a. Their union, $A \cup B$.

   b. The Cartesian product $A \times B$.

   c. The relative complement of B in A, written $A \setminus B$.

[3 points apiece]
2. Let $f : X \to Y$ be a function. Define the following:
   a. "f is one-to-one (injective)"

   b. "f is surjective (onto)"

   c. The *image* $f(A)$ [also written $f^{\to}A$] of a subset A of X

   d. The *preimage (inverse image)* $f^{\leftarrow}B$ [also written $f^{-1}(B)$] of a subset B of Y

[3 points]
3. Let $f : X \to Y$ be one-to-one and onto. Define the inverse function $f^{-1} : Y \to X$.

[3 points]
4. State the well-ordering principle on the set $\mathbb{N}$ of positive integers.

[3 points]
5. Tell what it means for r to be a rational number ($r \in \mathbb{Q}$).

[3 points each part]
6. Tell what it means for a subset B of $\mathbb{R}$ to be **bounded above** and tell what it means for r to be **the least upper bound (= supremum) of** B.

[6 points]
7. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 1$. Find $f^{\to}E$ and $f^{\leftarrow}E$ if $E = \{-1, 0, 1, 2, 3\}$

[2 points apiece]
8. Let $A = \{1, 3, 5, 7\}$, $B = \{1, 2, 6\}$, and $C = \{-1, 3, 6\}$. Find the following:

a. $A \cup B$

b. $A \cap (B \cup C)$

c. $A \setminus C$

d. $B \setminus (A \cup C)$

e. $B \times C$

[6 points]
9. Find the inverse of the function with domain $\mathbb{R} \setminus \{1\}$, defined by $f(x) = x/(x-1)$.

[6 points]
10. For A, B subsets of $\mathbb{R}$, define $A \cdot B = \{ab : a \in A, b \in B\}$. Let $A = \{-4, 0, 2, 3\}$ and $B = \{-2, 1, 2\}$. Find the following:

a. $A \cdot B$

b. $sup(A \cdot B)$

c. $(supA) \cdot (supB)$.

11. Let

$$f(x,y) = \begin{cases} x \text{ if } x = y \\ 0 \text{ if } x \neq y \end{cases}$$

Find the following:

[3 points apiece]

a. $F(x) = sup\{f(x,y) : y \in [2,4]\}$ for all $x \in [1,3]$.

b. $G(y) = sup\{f(x,y) : x \in [1,3]\}$ for all $y \in [2,4]$.

[2 points apiece]

c. $sup\{F(x) : x \in [1,3]\}$

d. $sup\{G(y) : y \in [2,4]\}$

[8 points]

12. Use mathematical induction fo show that $3^n > n^3$ for all $n \geq 4$, $n \in \mathbb{N}$.

[8 points]
13. Let $f : A \to B$ be a function. Show that $f^{\leftarrow}(A \cup B) = (f^{\leftarrow} A) \cup (f^{\leftarrow} B)$.

[5 points]
14. Assume that if $x, y \in \mathbb{R}$ and $0 \leq x < y$, there exists $q \in \mathbb{Q}$ such that $x < q < y$, and use this to show that there also exists such a $q \in Q$ if $x < 0$ and $x < y$.

[8 points]
15. Prove either one of the following. If you attempt both, you will be graded on the one you do better on.

a. If $x \in \mathbb{R}$ there exists $n \in \mathbb{N}$ such that $n \geq x$.

b. If $x, y \in \mathbb{R}$ and $x > 0$, there exists $n \in \mathbb{N}$ such that $nx \geq y$. You may assume part a.