CLASSIC PROBLEMS — 25 YEARS LATER (PART 2)

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The first article in this series [N02] (referred to below simply as "Part 1") dealt with the four Classic Problems that appeared in Volume 1 of this journal. This second article updates Part 1 and deals in detail with the four Classic Problems in Volume 2 and problems related to them.

The most progress on any of the eight problems this past year was made by Alan Dow on Classic Problem I ("Efimov's Problem"): Does every infinite compact space contain either a nontrivial convergent sequence or a copy of $\beta\omega$?

[As in Volume 2, "space" means "Hausdorff space."]

Alan Dow showed that there is a counterexample if $2^{\mathfrak{s}} < 2^{\mathfrak{c}}$ and the cofinality of the poset $([\mathfrak{s}]^{\omega}, \subset)$ is equal to $\mathfrak{s}$. Roughly speaking, Dow's construction substitutes zero-sets for points in Fedorchuk's PH construction [F77]. The construction can be done in ZFC, and results in an infinite compact space with no convergent sequences. The purpose of the second condition is to insure that the space has cardinality $2^{\mathfrak{s}}$, while the purpose of the condition $2^{\mathfrak{s}} < 2^{\mathfrak{c}}$ is to insure there is no copy of $\beta\omega$ in the space.

[Here $\mathfrak{s}$ stands for the splitting number. See [vD84] and [V90] for information on $\mathfrak{s}$ and other small uncountable cardinal numbers. Given a set S, the symbol $[S]^{\omega}$ stands for the set of all countably infinite subsets of S.]

The axiom $cf[\mathfrak{s}]^{\omega} = \mathfrak{s}$ is very general; its status is similar to that of the "small" Dowker space of Chris Good which is discussed below in connection with Classic Problem VII. That is, $cf[\mathfrak{s}]^{\omega} = \mathfrak{s}$ unless there is an inner model with a proper class of measurable cardinals. That is because $\mathfrak{s}$ is of uncountable cofinality, and because the Covering Lemma over any model of GCH is already enough to insure that $cf[\kappa]^{\omega} = \kappa$ for all cardinals *except* cardinals of countable cofinality. Now the Core Model satisfies GCH, and it is known that there is an inner model with a

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proper class of measurable cardinals whenever the Covering Lemma over the Core Model [abbreviated Cov(V,K)] fails.

The following well-known argument that $\mathfrak{s}$ is not of countable cofinality was pointed out by Heike Mildenberger. Suppose κ has cofinality ω, and no subcollection of $\mathcal{P}(\omega)$ of cardinality $< \kappa$ is splitting. Let $\mathcal{A}$ be a family of κ subsets of ω, and let $\mathcal{A} = \bigcup\{A_n : n \in \omega\}$ with $|\mathcal{A}_n| \le \kappa$ for all n. For each n, there is a set B_n that is not split by any member of $\mathcal{A}_n$ and which satisfies $B_{n+1} \subset B_n$. Then take an infinite pseudintersection of the B_n. This is a set that cannot be split by any member of $\mathcal{A}$.

A trivial modification of this argument shows that $cf(\mathfrak{s}) \ge \mathfrak{t}$. It is still not known whether $\mathfrak{s}$ is a regular cardinal.

The axiom that $2^{\mathfrak{s}} < 2^{\mathfrak{c}}$ is more restrictive, but still quite general. For example, given regular cardinals $\kappa < \lambda$, there is an iterated ccc forcing construction of a model where $\mathfrak{s} = \kappa$ and $\mathfrak{c} = \lambda$ [vD84, 5.1], where it is easy to see that the final model satisfies $2^{<\lambda} = \mathfrak{c}(< 2^{\mathfrak{c}})$. Even more simply, adding $\aleph_1$ Cohen reals to a model of $2^{\aleph_1} < 2^{\mathfrak{c}}$ results in a model where $\mathfrak{s} = \aleph_1$ and the other cardinals are not affected. Many other forcings have the same effect.

It might be worth mentioning here that Efimov's problem and Fedorchuk's constructions are of interest to analysts. Talagrand [T80] produced a Grothendieck space such that no quotient and no subspace contains ℓ_∞. A Banach space is called *Grothendieck* if every weak* convergent sequence in the dual space X^* is also weakly convergent. Talagrand's example was the Banach space $C(K)$ for a compact space K which contains neither $\omega + 1$ nor $\beta\omega$; it used CH for the construction.

A completely different application to analysis was done by Mirna Džamonja and Kenneth Kunen [DK]. They used $\diamondsuit$ to construct a compact S-space with no copy of either $\omega+1$ or $\beta\omega$, to give a hereditarily separable solution to the following problem: If X is compact and supports a Radon measure with nonseparable measure algebra, then does X map onto $[0,1]^{\omega_1}$? They were able to make the measure algebra isomorphic to the one for 2^{ω_1}.

1. After the last paragraph on page 364, add the following paragraph:

Piotr Koszmider has called my attention to a pair of Banach space equivalents to K having a copy of $\beta\omega$. One is that $C(K)$ (with the uniform topology) has ℓ_∞ as a quotient. The other is that $C(K)$ contains a subspace Banach-isomorphic to $\ell_1(\mathfrak{c})$. We do not know of conditions on $C(K)$ equivalent to K having a convergent sequence; a necessary condition is that $C(K)$ has a complemented copy of c_0.

A minor erratum in Part 1 was the claim that Balogh's first ZFC Dowker space [B89] was hereditarily realcompact; this is known only for another Balogh example [B01].

Here is an item related to Classic Problem IV, the M_3-M_1 problem: the paper by Mizokami, Shimane, and Kitamura has appeared [MSK]. In it, they prove that every WAP stratifiable space is M_1. A space X is said to be WAP iff for every non-closed A there is $x \in \overline{A} \setminus A$ and a subset B of A such that x is the only point in the closure of B which is not also in A. For example, sequential spaces are WAP and so are scattered spaces. It seems to be unknown whether $C_k(X)$ is WAP for all Polish X. In particular, it is unknown whether C_k(irrationals) is WAP, and it is still an open problem whether C_k(irrationals) is M_1; it is known to be M_3 [GR].

VOLUME 2

The first two of the Volume 2 problems are best considered together. Problem V is a double weakening of the more famous and older Problem VI:

Classic Problem V. Does every infinite compact hereditarily normal [abbreviated T_5] space of countable tightness contain a nontrivial convergent sequence?

Classic Problem VI. ("The Moore-Mrówka Problem") Is every compact space of countable tightness sequential?

In hindsight, Problem V may seem too specialized to be called a "classic." However, back in 1978 we were very much in the dark as to how well behaved compact spaces of countable tightness or compact T_5 spaces might be under ZFC-compatible axioms. Back then, we could not rule out the possibility that ZFC is enough to give a negative solution to Problem VI while Problem V is ZFC-independent. Also, we had no idea how long we would have to wait for a final solution to Problem VI even if it is ZFC-independent, and I felt that Problem V might give us a more attainable goal to shoot for in the interim.

We did have Fedorchuk's sensational 1975 construction under Axiom Φ [later shown equivalent to $\diamondsuit$] of an infinite compact T_5 hereditarily separable, hence countably tight space with no nontrivial convergent sequences, so we knew a negative solution to both problems is consistent. But the PFA, which turned out to imply a positive solution to Problem VI (and hence to V) had not even been formulated yet. The strongest general tool at our disposal in that direction was MA $+ \neg$CH; and that is actually compatible with a negative solution to Problem VI [N88]. Even now, it is still not known whether MA $+ \neg$CH is compatible with a negative solution to Problem V. Also, while we now know that a positive solution to Problem V is compatible with CH, the status of Problem VI under CH is still unsolved [E03] despite its being on the list of 26 unsolved problems in [A78]. [The statement in Volume 2 that Rajagopalan had constructed a compact non-sequential space of countable tightness from CH was incorrect.]

As it turned out, the solution to Problem V only predated the one for VI by a couple of months; but it could easily have been otherwise. The PFA solution to

Problem V was the culmination of five months of intensive research by by David Fremlin and myself beginning in March of 1986. We were working from combinatorial axioms derived from Martin's Maximum, which we soon narrowed down to one [N88, 6.8] that is now known to follow from the PFA, and does not require large cardinals. One discovery by Fremlin led to another by myself, which in turn led to new discoveries by Fremlin (some of which appear in [F88]). This continued until, on the way to the 1986 Prague International Topological Symposium, I showed that this axiom implies that every compact T_5 space of countable tightness is sequential [N88]. In Prague, I gave a copy of my proof to Zoltán Balogh. Fremlin and I continued to work on Problem VI and our joint efforts resulted in a proof that every compact space of countable tightness is sequentially compact under the PFA.

There the matter might have rested for a long time, had not Balogh meanwhile looked closely at Fremlin's proof that MM implies the axiom we were using, and thought "outside the box" as Gary Gruenhage put it last year when calling Balogh's solution to Problem VI the first of "Zoli's six greatest hits". Balogh did it by mixing topology into Fremlin's proof and coming up with a modification that even broke new set-theoretic ground. His solution came right at the end of 1986 and can be found in [B89]; a simplified version of the proof, using elementary submodels, can be found in [D92a]. Dow [D92b] later showed that large cardinals are not necessary for these applications of the PFA.

Related Problems for V and VI.

The biggest success story pertaining to any of the eight Classic Problems has to do with Problem V. Not only is the problem itself solved, but all those listed under the heading of *Related Problems* in Volume 2 have also been solved. These were:

Is every separable compact T_5 space
- (a) of countable tightness?
- (b) of cardinality $\leq \mathfrak{c}$?
- (c) sequentially compact?
- (d) sequential?

In Volume 2, it was explained how the axiom $2^{\aleph_0} < 2^{\aleph_1}$ gives a positive answer to (a) while Fedorchuk's construction under Axiom Φ ($\iff \diamondsuit$)[F76] gives negative answers to (b), (c), and (d). The PFA gives positive answers to all four parts [N92]. A model of MA $+ \neg$CH was given in [N92] where (a) is answered negatively.

To find a still-open problem in the discussion of Problem V in Volume 2, one has to look close to the end, where it is said, "It is not known whether every separable compact T_5 space is of cardinal $< 2^{\mathfrak{c}}$ under MA $+ \neg$CH." We do know from Jones's Lemma that $2^{|D|} \leq \mathfrak{c}$ for any discrete subset D of any separable T_5 space, and if we could substitute the Lindelöf degree of any subspace for $|D|$ when the space is compact, we would be done. However, Szentmiklóssy's theorem that every compact space of countable spread is hereditarily Lindelöf under MA $+\neg$CH does not generalize to arbitrary spreads $< \mathfrak{c}$. We also do not know of any model of MA $+\neg$CH where (b) or (c) has a negative answer, so we have only halfway met the challenge in the continuation of the above quotation: "In fact, it is a mystery what happens to any of these problems under MA $+\neg$CH." On the other hand, the final problem at the end of the discussion of Problem V in Volume 2 has been solved: MA $+\neg$CH is compatible with some version of $\gamma\mathbb{N}$ being T_5 [N92].

The *Related Problems* list for Problem VI ran:

A. Is there a hereditarily separable, countably compact, noncompact space?

B. (*Efimov*) Does a compact space of countable tightness have a dense set of points of first countability?

C. (*Hajnal and Juhász*) Is there a hereditarily separable compact space of cardinal $> \mathfrak{c}$?

D. Is there a compact space of countable tightess that is not sequentially compact?

E. Is every separable, countably compact space of countable tightness compact? What if it is locally compact?

F. (*Franklin and Rajagopalan*) Is every separable, first countable, countably compact [hence sequentially compact] space compact? What if it is locally compact?

All but the last two of these problems has been solved. In each of the other cases, Fedorchuk's Axiom Φ ($\iff \diamondsuit$) example [F76] solves the problem one way, while the PFA solves it the other way. In the case of Related Problem C, MA $+\neg$CH is enough to solve it in the other direction, as was already explained in Volume 2. In the case of Related Problem B, Malykhin showed that adding a single Cohen real is enough to produce a compact space X of countable tightness and π-character, in which every point of X has character ω_1 [M87]. In particular, if the ground model satisfies $\mathfrak{p} > \omega_1$ then X is Fréchet-Urysohn. Juhász [J88] showed that adding a single Cohen real results in a model where a weakening (t) of $\clubsuit$ holds, and that (t) is already enough to construct a space like Malykhin's.

The PFA solution to related problem A for regular spaces is due to Baumgartner and Todorčević, who showed that there are no S-spaces compatible with the PFA [B84], [T89]. Clearly, every countably compact noncompact space is non-Lindelöf and so a regular example for Related Problem A must be an S-space. For arbitrary (Hausdorff) spaces a slight modification of posets for the Moore-Mrówka problem [B89], [D92a] returns a negative PFA solution.

The PFA solution to Related Problem B is due to Alan Dow [D88], and the one to Related Problem D is due to Fremlin and myself as recounted above and in [N88]; the proof is similar to that of Statement 4 of [N92], but also uses free sequences of length ω_1 given by Statement D of [N92] to complete the "centrifugal saturation."

The status of Related Problems E and F is quite different from that of the others. There is a ZFC counterexample for the first part of Related Problem E [NV], but it is not even Urysohn, let alone Hausdorff. For regular spaces, almost all of what we know is already to be found in [N90], including the information that almost every known regular counterexample for Statement E is also a counterexample for Statement F; that almost every published counterexample is also locally compact; and that this is one of the growing list of problems for which there are counterexamples if $\mathfrak{c}$ is either $\aleph_1$ or $\aleph_2$: there are counterexamples both if $\mathfrak{p} = \aleph_1$ and if $\mathfrak{b} = \mathfrak{c}$, and the well-known fact that $\mathfrak{p} \leq \mathfrak{b}$ gives us no room for loopholes if $\mathfrak{c} \leq \aleph_2$.

Incidentally, Related Problem F is one of my personal favorites. At the 1986 Prague International Toplogical Symposium I offered a prize of 500 US Dollars for a solution, and raised it to 1000 at the 1996 Prague Toposym. Despite this, almost no progress has been made on it since 1986.

Classic Problem VII. Does there exist a "small" Dowker space? More precisely, does there exist a normal space which is not countably paracompact and is one or more of the following:

A. First countable?
B. [hereditarily] separable?
C. of cardinality $\aleph_1$?
D. submetrizable?
E. locally compact?

The word "small" is very informal and one person's list of properties might easily differ greatly from another's. Most people would probably agree that "of cardinality $\leq \mathfrak{c}$" has a greater claim to being called "small" than submetrizability or local compactness. Had I put it in, then the most significant advance on Problem VII in the last 25 years would arguably have been Balogh's ZFC example in [B96]. [Its main competitor, as ably explained in the introduction to [KS], would be a Dowker space shown in ZFC to be of cardinality $\aleph_{\omega+1}$.] As it is, the most significant is clearly Chris Good's construction of a locally compact, locally countable (hence first countable) Dowker space under a higher-cardinal analogue of $\clubsuit$ that follows from Cov(V,K) and hence requires very large cardinals for its negation [G95]. Good gave a general construction which also works under $\clubsuit$ to give an example that is, in addition, of cardinality $\aleph_1$. Moreover, it can be embedded in a separable example using the technique Peter deCaux used at the end of his paper for his very similar example [dC76].

Good used consequences of Cov(V,K) similar to those employed by Bill Fleissner for his solution of the bigger half of the normal Moore space problem [F84]. The smallest examples in either case have cardinality $\beth_\omega^+$. This is the successor of the first singular strong limit cardinal, $\beth_\omega$, which is the supremum of the sequence of cardinals $\beth_n$ where $\beth_0 = \aleph_0$ and $\beth_{n+1} = 2^{\beth_n}$.

Like de Caux's example, Good's examples are all countable unions of discrete subspaces. However, they are not submetrizable. On the other hand, the second example in [JKR] is submetrizable, as mentioned in Volume 2 already.

As already recounted in Volume 2, there is a construction of a Dowker space from CH that satisfies all but the last part of Classic Problem VII. See [JKR], where a $\diamondsuit$ construction was announced that satisfies all five conditions simultaneously, including the hereditary version of condition B. This does not seem to have ever appeared in print, but there is a $\diamondsuit$ construction in [J77] that satisfies all the conditions except D, submetrizabilty. One erroneous comment from [JKR] carried over to the Volume 2 discussion. It was claimed that the $\diamondsuit$ example is σ-countably compact, but there is no such thing as a σ-countably compact Dowker space.

We still do not have a locally compact Dowker space from CH alone. On the other hand, I know of only two independence results directly bearing on Problem VII as stated. One is that there is no first countable, locally compact, submetrizable example of cardinality $\aleph_1$ under MA $+ \neg$CH. This is because of Balogh's theorem [B83] that under MA $+ \neg$CH, every first countable, locally compact space of cardinality $\aleph_1$ either contains a perfect preimage of ω_1 (hence cannot be submetrizable) or is a Moore space. Now, Moore spaces are countably metacompact, and normal spaces are countably paracompact iff they are countably metacompact.

The other independence theorem has little to do with Dowker-ness. The "hereditarily" version of part B is consistently false because the PFA implies that there are no S-spaces [B84] [T89] and so every hereditarily separable space is Lindelöf and therefore (countably) paracompact. In contrast, the PFA actually implies the existence of first countable Dowker spaces, and is consistent with the existence of first countable, locally compact Dowker spaces [R84]: Murray Bell's first countable example [B81] exists under $\mathfrak{p} = \mathfrak{c}$, which is implied by MA $+ \neg$CH and hence by the PFA; and Weiss constructed a locally compact first countable example assuming $\mathfrak{p} = \mathfrak{c} = \omega_2 + \diamondsuit_{\mathfrak{c}}(\mathfrak{c}, \omega\text{-limits})$ [W81] [R84], and this combination of axioms is known to be compatible with the PFA. There are also examples of first countable Dowker spaces of cardinality $\aleph_1$ compatible with the Product Measure Extension Axiom (PMEA) [G95].

Despite all this, we seem very far from any ZFC examples, except perhaps for Part D of Problem VII. At the beginning of April, 2002, less than four months before his death, I sent Zoltán Balogh an e-mail in which I asked him whether any of his Dowker examples were submetrizable. In his reply, which came the same day, he wrote,

"One of my Dowker space is almost submetrizable, and I somehow thought it could be made submetrizable. Give me a couple of weeks on that and I'll let you know."

That was the last I ever heard from him. Part D of Problem VII remains unsolved as far as we know.*

The *Related Problems* list for Problem VII ran:

(1) Is there a pseudonormal space (a space such that two disjoint closed subsets, one of which is countable, are contained in disjoint open sets) which is not countably metacompact, and which is one or more of the above [A, B, C, D, E]?

(2) Is there a realcompact Dowker space?

(3) Is there a monotonically normal Dowker space?

The answers to (2) and (3) are "Yes" [B01] and "No" [R84], respectively. As for (1), there is a ZFC example of a 2-manifold which is pseudonormal but not countably metacompact in [N84]. Like all manifolds, it is locally compact and first countable (A & E). It is produced by adding half-open intervals to the open first octant in the square of the long line. A routine modification of the topology on the subspace of those points with ordinal coordinates, together with endpoints of the added intervals produces a first countable, locally compact pseudonormal space of cardinality $\aleph_1$ which is still not countably metacompact. Finally, this subspace can be embedded in a separable example like Good's example, still in ZFC, giving A & B^- & C & E.

I am unaware of any submetrizable (Part D) examples just from ZFC. Locally compact, first countable, submetrizable ones of cardinality $\aleph_1$ (A & C & D & E) are ruled out just as they are for Dowker spaces. So too are hereditarily separable examples (B^+).

Classic Problem VIII. Is every γ-space quasi-metrizable?

The answer is "No". Ralph Fox [F82] came up with a machine which outputs a γ-space with each γ-space input, and which produces non-quasi-metrizable spaces in certain cases. It preserves the Hausdorff separation axiom, but not regularity. Together with Jacob Kofner, Fox found a Tychonoff example [FK] which is quasi-developable and scattered. In a note added in proof to [FK], they announced the construction of a paracompact γ-space that is not quasi-metrizable. Now, Hans-Peter Künzi has done us the service of publishing a description of the example and an outline of the proof that it works [K01].

The *Related Problems* list for Problem VIII ran:

(1) Is every paracompact (or Lindelöf) γ-space quasi-metrizable?

(2) Is every γ-space with an ortho-base quasi-metrizable?

(3) Is every linearly orderable γ-space quasi-metrizable?

The answer to the "paracompact" part of (1) is "Yes" as recounted above. For "Lindelöf" it is still open. We also do not have a ZFC example of a Lindelöf γ-space that is not non-Archimedeanly quasi-metrizable. A Luzin subset of the Kofner plane [K73] [K80b, Example 1] is a consistent example: see Proposition 5 in [K80a], which was mis-stated with the omission of "not" before "non-Archimedean".

Kofner also provided affirmative answers to (2) [K81b] and (3) [K81a] (3). In both cases, Kofner used the fact that every k-transitive γ-space is non-Archimedeanly quasi-metrizable, for any integer k. The former proof uses the fact that any space with an ortho-base is 2-transitive, while the latter uses the fact that every GO-space is 3-transitive. His article [K80] for Topology Proceedings is a very nice survey of the state of the art at the time.

Added in Proof. As the galleys for this article were being prepared, Dennis Burke found some handwritten notes by Zoltán Balogh dated 4/25/02-5/1/02 in which he seems to be describing a ZFC example of a submetrizable Dowker space. It is too early to tell from the notes whether the example is correct.

References

[A78] A. V. Arhangel'skiĭ, "The structure and classification of topological spaces and cardinal invariants," *Russian Math. Surv.* **33** (6) (1978) 33–96.

[B81] M. Bell, "On the combinatorial principle P($\mathfrak{c}$)," *Fund. Math.* **114** (1981) 137–145.

[B83] Z. Balogh, "Locally nice spaces under Martin's Axiom," *Comment. Math. Univ. Carolinae* **24** (1983), 67–87.

[B84] J. Baumgartner, "Applications of the Proper Forcing Axiom," *Handbook of Set-Theoretic Topology,* K. Kunen and J. Vaughan ed., North-Holland (1984) 913–959.

[B89] Z. Balogh, "On compact Hausdorff spaces of countable tightness," *Proc. AMS* **105** (1989) 755-764. MR 89h:03088

[B96] Z. Balogh, "A small Dowker space in ZFC," *Proc. AMS* **124** (8) (1996), 2555–2560. MR 96j:54026

[B01] Z. Balogh, "Dowker spaces and paracompactness questions," *Top. Appl.* **114** (1) (2001), 49–60. MR 2003c:54047

[dC76] P. deCaux, "A collectionwise normal, weakly θ-refinable Dowker space which is neither irreducible nor realcompact," *Topology Proc.* **1** (1976) 66–77.

[D88] A. Dow, "An introduction to applications of elementary submodels to topology," *Topology Proc.* **13** (1988), no. 1, 17–72.

[D92a] A. Dow, "Set theory in topology," in: *Recent Progress in General Topology,* M. Hušek and J. van Mill, eds., Elsevier, 1992, 167–197.

[D92b] A. Dow, "On the consistency of the Moore-Mrówka solution," Top. Appl. 44 (1-3) (1992), 125–141.

[DK] M. Džamonja and K. Kunen, "Measures on compact HS spaces," *Fund. Math.* **143** (1) (1993) 41–54.

[vD84] E.K. van Douwen, "The integers and topology," *Handbook of Set-Theoretic Topology,* K. Kunen and J. Vaughan ed., North-Holland (1984) 111–167.

[E03] T. Eisworth, "Totally proper forcing and the Moore-Mrówka problem," *Fund. Math.* **177** (2003) 12—137.

[F76] V. V. Fedorchuk, "Fully closed mappings and the consistency of some theorems of general topology with the axioms of set theory," *Math. Sbornik* **99** (141) (1976) = *Math. USSR Sbornik* **28** (1976) 1–26.

[F77] V. V. Fedorchuk, "A compact space having the cardinality of the continuum with no convergent sequences," *Math. Proc. Cambridge Phil. Soc.* **81** (1977) 177–181.

[F82] R. Fox, "Solution of the γ-space problem," *Proc. AMS* **85** (4) (1982) 606–608. MR 83h:54035

[F84] W. G. Fleissner, "The Normal Moore Space Conjecture and Large Cardinals," in: *Handbook of Set-Theoretic Topology*, K. Kunen and J. Vaughan ed., North-Holland, 1984, pp. 733–760.

[F88] D.H. Fremlin, "Perfect pre-images of ω_1 and the PFA," *Top. Appl.* **29** (2) (1988) 151–166.

[FK] R. Fox and J. Kofner, "A regular counterexample to the γ-space conjecture," *Proc. AMS* **94** (3) (1985) 502–506. MR 86m:54033

[G95] C. Good, "Large cardinals and small Dowker spaces," *Proc. Amer. Math. Soc.* **123**, no. 1 (1995) 263–272.

[GR] P. M. Gartside and B. A. Reznichenko, "Near metric properties of function spaces," *Fund. Math.* **164** (2000), 97–114. MR 2002b:54026

[J77] I. Juhász, "Consistency results in topology," in: *Handbook of Mathematical Logic*, J. Barwise ed., North-Holland (1977) 503–522.

[J88] I. Juhász, "A weakening of $\clubsuit$, with applications to topology" *Comment. Math. Univ. Carolinae* **29** (1988), no. 4, 767–773.

[JKR] I. Juhász, K. Kunen, and M.E. Rudin, "Two more hereditarily separable non-Lindelöf spaces," *Canad. J. Math.* **28** (1976) 998–1005.

[K73] J. (Ja.A) Kofner, "Δ-metrizable spaces," *Mat. Zametki* **13** (1973), 277–287 = *Math. Notes* **13** (1973), 168–174.

[K80a] J. Kofner, "Quasi-metrizable spaces," *Pacific J. Math.* **88** *(1) (1980), 81–89.*

[K80b] *J. Kofner, "On quasi-metrizability," Topology Proc.* **5** *(1980) 111–138. MR 82j:54054*

[K81a] *J. Kofner, "Transitivity and the γ-space conjecture," Proc. AMS* **81** *(4) (1981) 629–635. MR 82h:54050*

[K81b] *J. Kofner, "Transitivity and ortho-bases," Canad. J. Math.* **33** *(6) (1981) 1439–1447. MR 83c:54039*

[K01] *H.-P.A. Künzi, "Nonsymmetric distances and their associated topologies: about the origins of basic ideas in the area of asymmetric topology," Handbook of* *the History of General Topology, Volume 3, C.E. Aull and R. Lowen eds., Kluwer Academic Publishers, Netherlands, 2001, pp. 853-968.*

[KS] *M. Kojman and S. Shelah, "A ZFC Dowker space $\aleph_{\omega+1}$: an application of pcf theory to topology," Proc. AMS* **126** *(8) (1998), 2459–2465.*

[M87] *V. I. Malykhin, "A Fréchet-Urysohn compact set without points of countable character" (in Russian), Mat. Zametki* **41** *(1987), no. 3, 365–376, 457; MR 88g:54045*

[MSK] *T. Mizokami, N. Shimura, and Y. Kitamura, "A characterization of a certain subclass of M_1-spaces," JP J. Geom. Topol.* **1** *(2001) 37–51.*

[N84] *P. Nyikos, "The theory of nonmetrizable manifolds," Handbook of Set-Theoretic Topology, K. Kunen and J. Vaughan ed., North-Holland (1984) 633–684.*

[N88] *P. Nyikos, "Progress on countably compact spaces" in: General Topology and its Relations to Modern Analysis and Algebra VI: Proc. Sixth Prague Topological Symposium 1986, Z. Frolík ed., Berlin, Heldermann Verlag, 1988, pp. 379–410.*

[N90] *P. Nyikos, "On first countable, countably compact spaces III: The problem of obtaining separable noncompact examples," Open Problems in Topology, J. van Mill and G. M. Reed ed., North-Holland (1990) pp. 127–161.*

[N92] *P. Nyikos, "Hereditary normality versus countable tightness in countably compact spaces," Top. Appl.* **44** *(1992) 271–292.*

[N98] *P. Nyikos, "The structure of locally compact T_5 spaces under strong axioms," Topology Proceedings* **23***, Spring 1998, pp. 349-356.*

[N02] *P. Nyikos, "Classic Problems — 25 Years Later (Part 1)," Topology Proceedings* **26** *(1) (2002) 345–356.*

[NV] *P. Nyikos and J. Vaughan, "The Scarborough-Stone problem for Hausdorff spaces," Top. Appl.* **44** *(1992) 309–316.*

[R84] *M. E. Rudin, "Dowker Spaces", in: Handbook of Set-Theoretic Topology, K. Kunen and J. Vaughan ed., North-Holland (1984) 761–780.*

[T80] *M. Talagrand, "Un nouveau $\mathcal{C}(K)$ qui posède la propriété de Grothendieck," Israel J. Math.* **37** *(1-2) (1980) 181–191.*

[T89] *S. Todorčević, "Partition problems in topology," Contemporary Mathematics* **84***, American Mathematical Society, 1989.*

[V90] *J. Vaughan, "Small uncountable cardinals and topology," in: Open Problems in Topology, J. van Mill and G. M. Reed ed., North-Holland (1990) pp. 195–218.*

[W 81] *W. Weiss, "Small Dowker spaces," Pac. J. Math.* **94** *(1981) 485–492.*