

Name _____

Seat # _____

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student’s calculator during the test.
- Show your Student ID when you turn in your test.

Do not write below this line

#1 (5 points) _____ #7 (15 points) _____

#2 (5 points) _____ #8 (15 points) _____

#3 (5 points) _____ #9 (15 points) _____

#4 (5 points) _____ #10 (5 points) _____

#5 (5 points) _____ #11 (10 points) _____

#6 (10 points) _____ #12 (5 points) _____

TOTAL (100 points) _____

Test 1 _____ Test 2 _____ Test 3 _____ Total _____

If you skip the final exam, your course grade will be _____

1. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dq}{dv} = 3 \quad \text{and} \quad q(0) = 60$$

2. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dp}{dt} = 10t \quad \text{and} \quad p(0) = 20$$

3. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dw}{dr} = 2w \quad \text{and} \quad w(0) = 30$$

4. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dy}{dx} = -8e^{-2x} \quad \text{and} \quad y(0) = 25$$

5. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dh}{dt} = \frac{e^t}{3h^2} \quad \text{and} \quad h(0) = 2$$

6. (10 points) Suppose that a fish population grows logistically with an intrinsic growth rate of 25% and a carrying capacity of 800.

(a) Determine a discrete dynamical system to model this fish population.

(b) Determine the maximum interval of stability for this fish population.

7. (15 points) For John's metabolism, the dynamical system modeling the elimination of alcohol is given by

$$a(n) = a(n - 1) - \frac{10a(n - 1)}{4 + a(n - 1)} + d$$

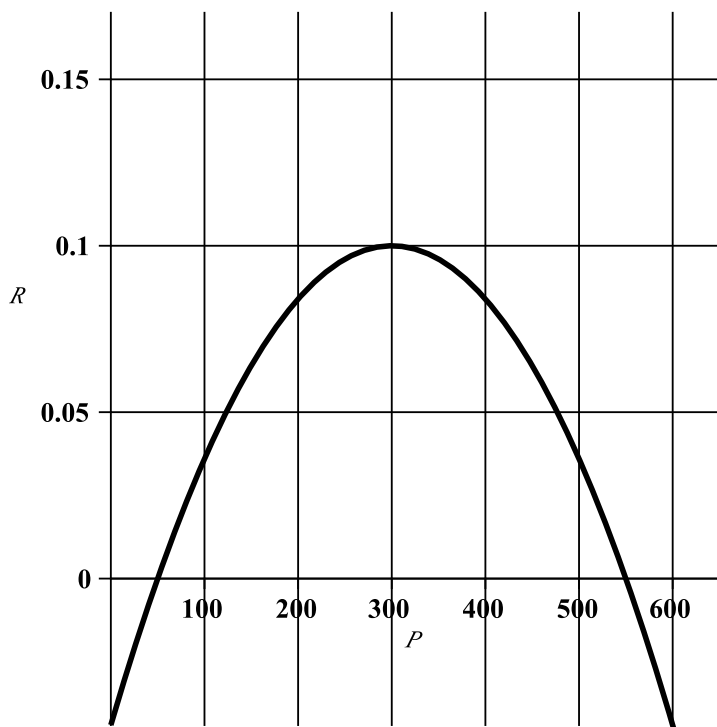
where $a(n)$ is the amount of alcohol (in grams) in his bloodstream after n hours of drinking d grams of alcohol per hour.

- (a) If John were to drink 28 grams of alcohol per hour, how much alcohol would be in his bloodstream after drinking this amount for 4 hours? Begin with $a(0) = 0$.
- (b) How many grams of alcohol per hour can John drink if at the end of a 4 hour party he is to have 40 grams of alcohol in his bloodstream? Begin with $a(0) = 0$ and give your answer correct to one place after the decimal.
- (c) Compute the equilibrium amount of alcohol in John's bloodstream if he drinks 9.5 grams of alcohol per hour.

8. (15 points) A population P can be modeled by the discrete dynamical system

$$P(t) = P(t - 1) + R \cdot P(t - 1)$$

where the growth rate R is a function of P as shown in the graph below.



(a) Which one of the following is the intrinsic growth rate for this population?

- (A) 1% (B) 3.33% (C) 5% (D) 10% (E) 15% (F) 30%

(b) Which one of the following populations is a stable equilibrium value?

- (A) 0 (B) 50 (C) 100 (D) 300 (E) 550 (F) 600

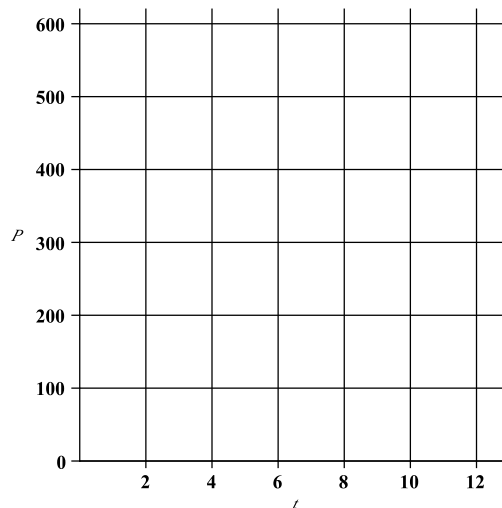
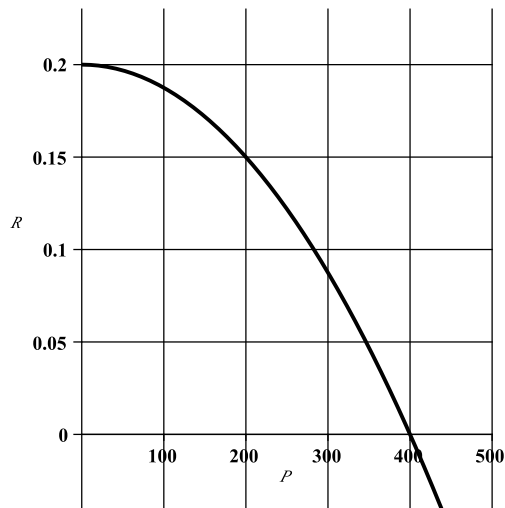
(c) Which one of the following is the minimum viable population?

- (A) 0 (B) 50 (C) 100 (D) 300 (E) 550 (F) 600

9. (15 points) A population P can be modeled by the discrete dynamical system

$$P(t) = P(t - 1) + R \cdot P(t - 1)$$

where the growth rate R is a function of P as shown in the graph on the left.



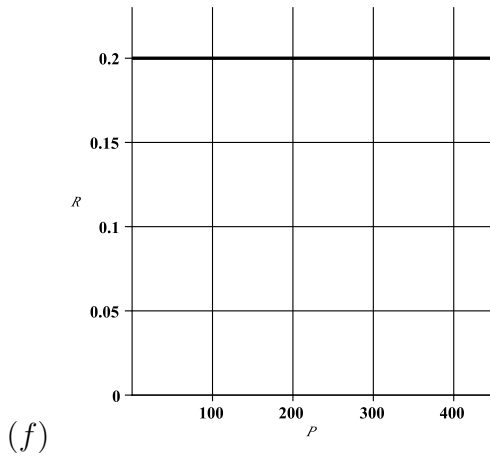
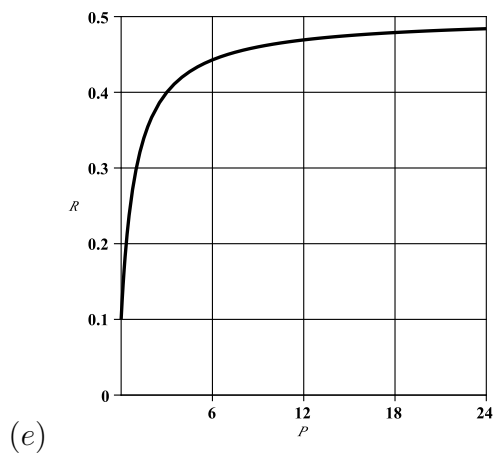
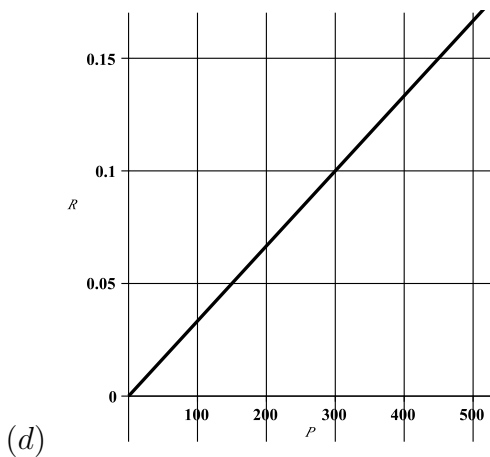
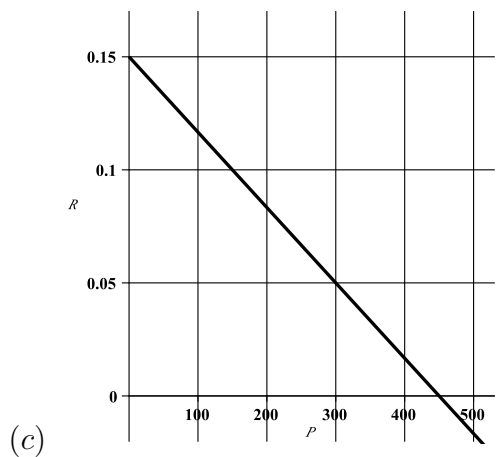
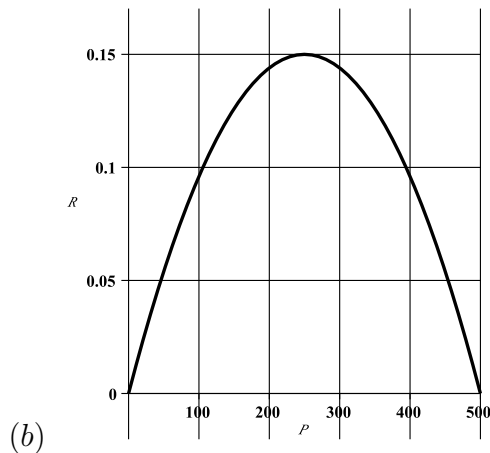
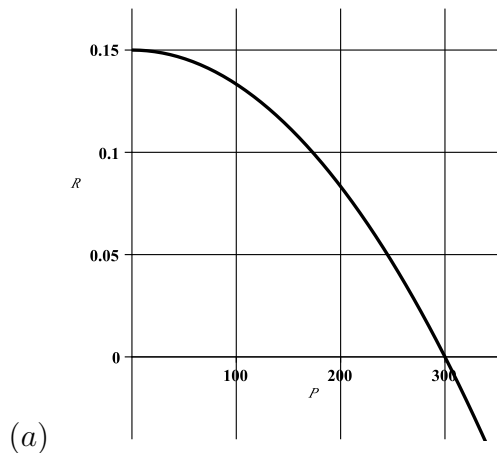
- (a) Given that $P(0) = 200$, use the grid on the right to sketch a rough graph of the population as a function of time. The time scale shown is sufficiently long that any long term behavior for the population should be apparent from your graph.
- (b) Determine a formula for R in terms of P given that the graph of R is a parabola with vertex $(0, 0.2)$.

- (c) Compute the value of $P(1)$ given that $P(0) = 200$.

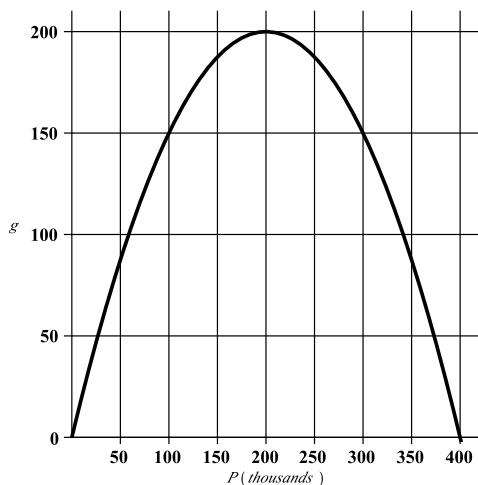
10. (5 points) A population grows logistically and is modeled by the following discrete dynamical system

$$P(t) = P(t - 1) + R \cdot P(t - 1)$$

where R represents the growth rate and is a function of the population P . Which one of the following graphs is the only one that could possibly be the graph of R as a function of P ?

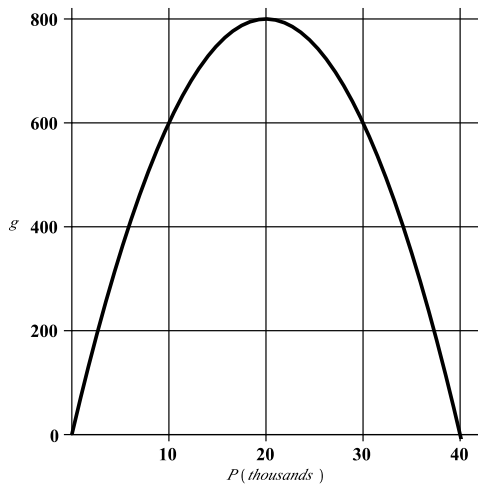


11. (10 points) The natural yearly growth g in a fish population is a function of the population size P (in thousands) and is shown in the following graph. Suppose that there is a constant yearly harvest of 150 fish.



- (a) Estimate the stable equilibrium population.
- (b) Estimate the minimum viable population.

12. (5 points) The natural yearly growth g in a population is a function of the population size P (in thousands) and is shown in the following graph.



What percent of the population should be harvested each year if you wish to maximize the eventual yearly harvest?